YieldCurve.com

YIELD CURVE FITTING 2.0

Constructing Bond and Money Market Yield Curves using Cubic B-Spline and Natural Cubic Spline Methodology

Users' Manual

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YieldCurve.Com

Version 2.0

March 2005

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Introduction

Welcome to the Yield Curve Fitting (YCF) version 2.0 manual. YCF is an Excel-based program that calculates discount factors, zero-coupn interest rates and forward rates using bond market, money market and interest rate swap input data.

YCF has three sheets, which are MainPage, BondMarket, and MoneyMarketAndSwaps respectively. In the BondMarket sheet, users calculate various yield curves using bond market data by means of the Cubic B-Spline Fitting Methodology. In the MoneyMarketAndSwaps sheet, users calculate various yield curves either using pure money market data through Natural Cubic Spline Interpolation or combining with interest rate swap data by the Bootstrap method. Users can access BondMarket or MoneyMarketAndSwaps via the MainPage.

System requirements:

- Windows NT/2000/XP
- Microsoft Excel 2000 or higher
- Acrobat Reader 5.0 or higher

Chapter 1

YCF 2.0 User Interface

Main Page

The Main Page is shown as follows.

- Click "help" in the left upper corner to view the help manual.
- Click "Bond Market " link to access BondMarket page.
- Click " Money Market & Swaps" link to access MoneyMarketAndSwaps page.

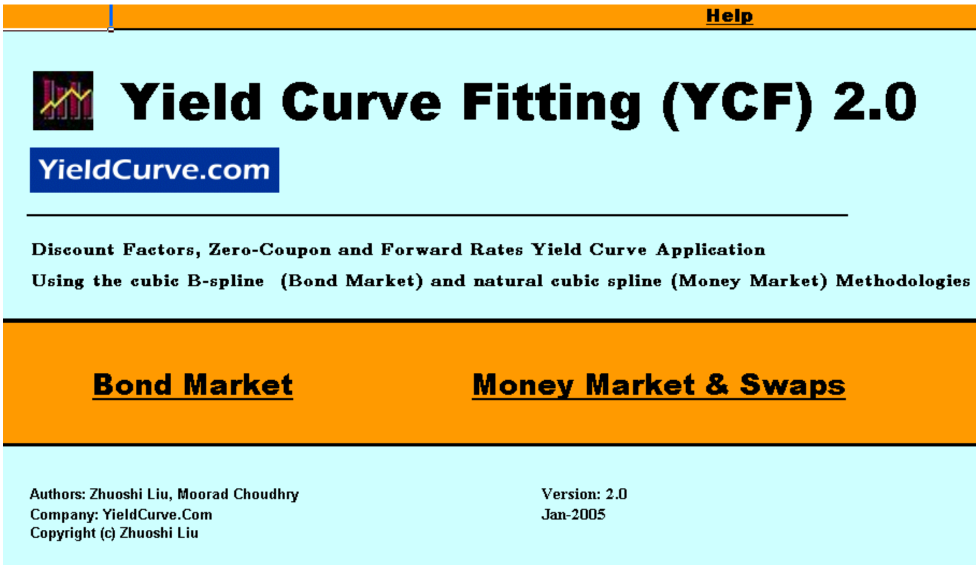


Fig 1

Bond Market

The BondMarket page has 3 parts. These are Input Initial Parameters Section (shown in Fig 2), Input Coupon Bond Market Data Section (shown in Fig 2), and Yields Curve Output Section (shown in Fig 3).

A. Input Initial Parameters Section

Users input Settlement Date, Results/Forward Rates Frequency, Input Day-Count Basis and Output Day-Count Basis in this section.

Settlement Date: The value date on which bonds yields are quoted. The input date format is: dd-mmm-yyyy , e.g. 23-Mar-2005.

Results/Forward Rates Frequency: This is the frequency for both forward rates and results display. For example, if you choose "Monthly":

The calculated yield data will be displayed in a monthly frequency and the forward rates are calculated using monthly discount factors or zero rates.

Input Day-Count Basis: Day Count-Basis used in calculating discount factors from coupon bond data.

Output Day-Count Basis: Day Count-Basis used in calculating zero-coupon rates and forward rates from the discount factors.

B. Input Coupon Bond Market Data Section

Users input Coupon bond data in this section, which includes:

Maturity Date: The date on which the bond will mature. The input date format is: dd-mmm-yyyy , e.g. 23-Mar-2005.

Coupon Rates: Indicating the annual percentage rate used to determine the coupons payable on a bond. For a 5% coupon rate, please input "5", instead of say, "0.05".

Coupon Frequency: Coupon payment frequency. Users have three options to choose from, Quarterly, Semi-annual and Annual coupon payment.

Yields: Bond yield to maturity on a semi-annual basis. For a 5% bond yield, please input "5", instead of say, "0.05".

When users finish inputting coupon bond data, they can proceed by clicking the button below.

> **Calculate Curve**: Click it, and you will get the calculated results for the discount factors, zero-coupon rates, and forward rates.

If you want to clear the old coupon bond data from the table, please click the button below.

> **Clear Table**: Click it, and you will clear all the data in the table except "Coupon Frequency" which are now reset to "semi-annual".

Notes for bond data input

1. Please input bond yields in their maturity date order.
2. Click the calendar button to choose the maturity date.
3. Please input "0" if the bond has no coupon.
4. You cannot use the "Calculate Curve" button while the cursor still focuses in the cell.

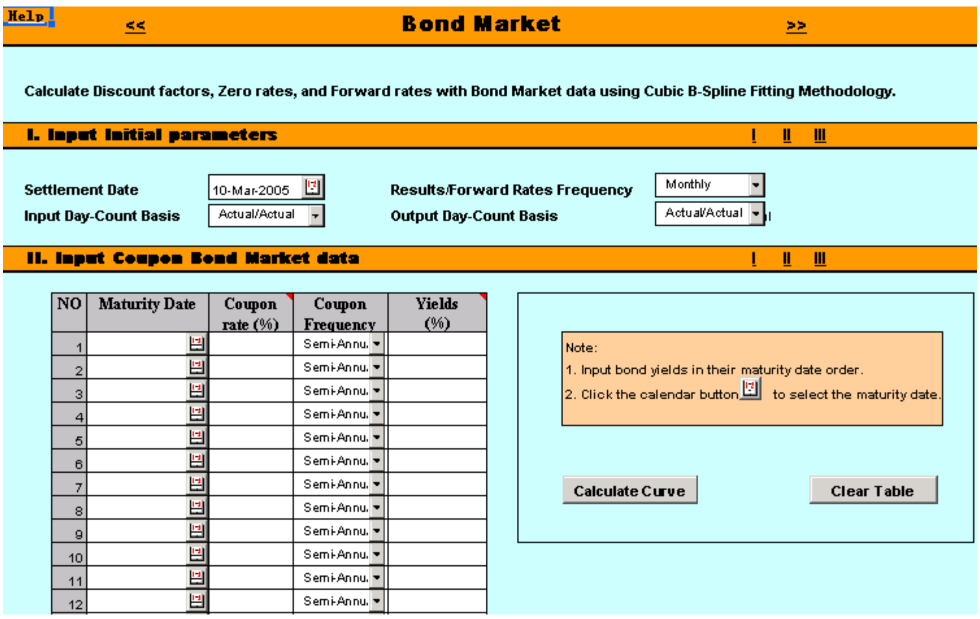


Fig 2

C. Yields Curve Output Section

After click “Calculate Curve” button, the spreadsheet automatically moves the focus to this section, and displays the following yield curve output:

> **Discount factor**: The ratio of the present value of a financial asset to its future value.
>
> **Zero Rate**: The annual interest rates that would be earned on a bond that provides no coupons.
>
> **Forward Rate**: The annual interest rate for a future period of time that are implied by the discount factors that prevail today.
>
> **YTM**: Annual yield to maturity of bonds.

The frequency of the yields (and discount factors) shown above is determined by the choice of “Results/Forward Rates Frequency” in the “Input Initial Parameters Section”.

You can plot the various curves by clicking following buttons.

> **Plot Discount Factors**: Click it, and you will get the discount curve graph.
>
> **Plot Other Curves**: After you click it, a popup window will appear and ask you which yield curve you want plot. You can choose to plot any yield curve in the same graph by checking them. The yield curves you can plot are bond yield to maturity curve, zero-coupon curve, and forward rates yield curve.

You can calculate curve data for any selected value date. To do this, first select value date, and then choose the type of the curve, finally click the “Calculate” button to get the result.

Value date can be any dates from the earliest maturity date to the latest maturity date that are displayed in the output table. Its format is “dd-mmm-yyyy”

Click “Clear Contents” button to clear all the output data in the table. This is not an obligatory step.

Notes for the yield curve output Section

1. Please make sure that you have curve data in the output table before plotting curves or calculating the curve data for any selected value date.

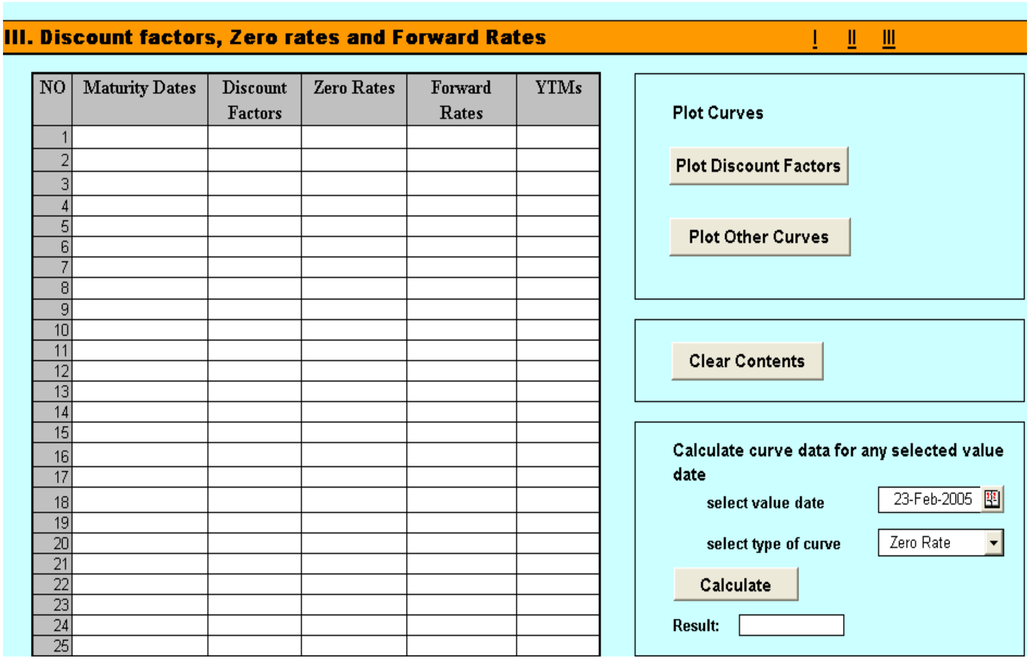


Fig 3

Tips for the BondMarket Spreadsheet

There are convenient links on the right hand side of each orange band. By clicking "I", "II" or "III", you can go to Section I, Section II or Section III. Due to the large size of the spreadsheet, this feature is very helpful.

Money Market and Swaps

MoneyMarekt&Swaps page includes 4 parts. They are Input Initial Parameters Section (shown in Fig 4), Input Money Market Rates Section (shown in Fig 4), Input Swap Rates Section (shown in Fig 5), and Yields Curve Output Section (shown in Fig 6).

A. Input Initial Parameters Section

Users input Settlement Date, Results/Forward Rates Frequency, and Output Day-Count Basis in this section.

Settlement Date: The value date on which bonds yields are quoted. The input date format is: dd-mmm-yyyy , e.g. 23-Mar-2005.

Results/Forward Rates Frequency: This is the frequency for both forward rates and results display. For example, if you choose "Monthly", the calculated yield data will be displayed in a monthly frequency and the forward rates are calculated using monthly discount factors or zero rates.

Output Day-Count Basis: Day Count-Basis used in calculating zero-coupon rates and forward rates from the discount factors.

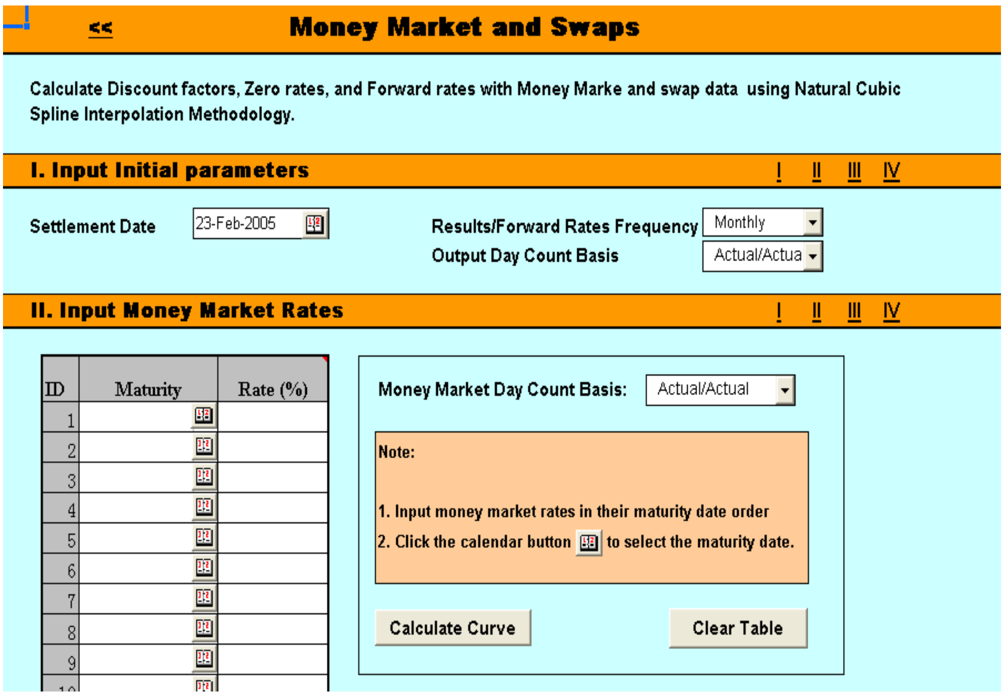


Fig 4

B. Input Money Market Rates Section

Users input money market data and related parameters in this section, which including:

Maturity Date: The date on which the money market instruments will mature. The input date format is: dd-mmm-yyyy , e.g. 23-Mar-2005.

Rates: Indicating the annual percentage return of the money market instruments. For a 5% rate, please input "5", instead of "0.05".

Money Market Day-Count Basis: Day Count-Basis used in calculating the discount factors from the money market rates.

Users can calculate yield curves with money market data only. In this case, users should click the button below to proceed.

Calculate Curve: Click it, and you will get the estimated results for the discount factors, zero rates, and forward rates.

If you want to estimate yield curves with money market rates and swap rates, simply move on to the next section to input the swap rates.

If you want to clear the old money market data from the table, please click the button below.

Clear Table: Click it, and you'll clear all the data in both money market rates table and swap rates table.

Notes for money market data input

1. Please input money market rates in their maturity date order.
2. Click the calendar button  to choose the maturity date.
3. You cannot use the "Calculate Curve" button while the cursor still focuses in the cell.

C. Input Swap Rates Section

Users input interest rate swaps data and related parameters in this section, which includes:

Maturity Date: The date on which the interest rate swaps will mature. The input date format is: dd-mmm-yyyy , e.g. 23-Mar-2005.

Rates: The annual swap rates. For a 5% rate, please input "5", instead of "0.05".

Swap Frequency: This is the frequency of the cash exchange between fixed interest payment and floating interest payment for a swap agreement.

Swap Day-Count Basis: Day Count-Basis used in calculating the discount factors from the swap rates.

After users input both money market rates and swap rates, they can proceed by clicking the button below.

Calculate Curve: Click it, and you will get the estimated discount factors, zero rates, and forward rates.

If you want to clear the old swap data from the table, please click the button below.

Clear Table: Click it, and you'll clear all the data in both money market rates table and swap rates table.

Notes for swap rates input

1. Please input swap rates in their maturity date order.
2. Please make sure you have also input money market data before you calculate yield curves.
3. The first swap maturity date must be earlier than or equal to the last money market rate maturity.
4. You cannot use the "Calculate Curve" button while the cursor still focuses in the cell.

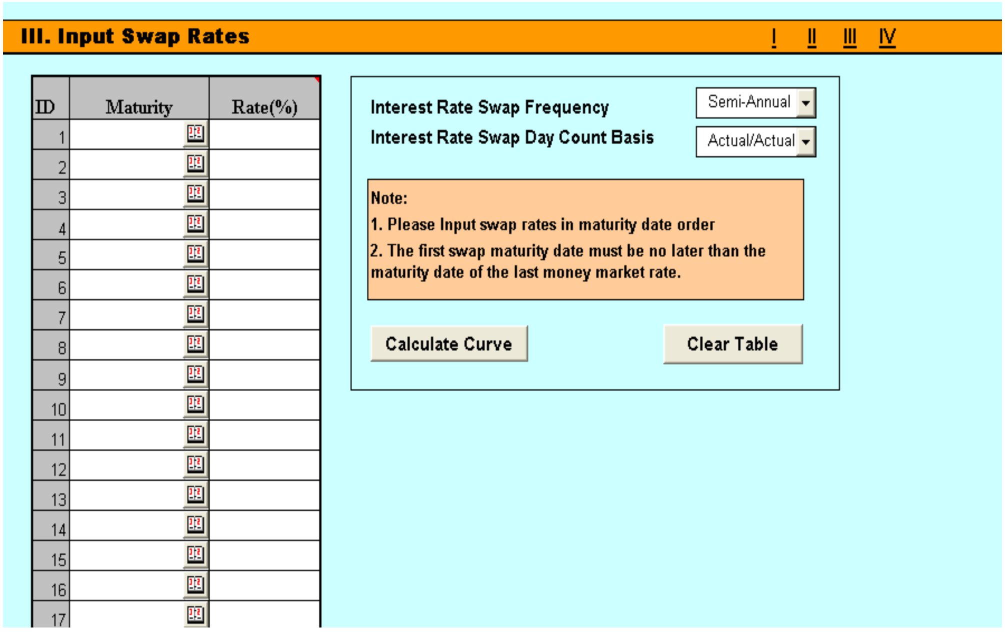


Fig 5

D. Yields Curve Output Section

After click "Calculate Curve" button, the spreadsheet automatically moves the focus to this section, and displays the following yield curve output:

Discount factor: The ratio of the present value of a financial asset to its future value.

Zero Rate: The annual interest rates that would be earned on a bond that provides no coupons.

Forward Rate: The annual interest rate for a future period of time; these are implied by the discount factors that prevail today.

The frequency of the yields (and discount factors) shown above is determined by the choice of "Results/Forward Rates Frequency" in the "Input Initial Parameters Section".

You can plot the various curves by clicking following buttons.

Plot Discount Factors: Click it, and you will plot the discount curve.

Plot Other Curves: After click it, a popup window will appear and ask you which yield curve you want plot. You can choose to plot any yield curve in the same graph by checking them. The yield curves you can plot are zero-coupon curve and forward rates yield curve.

You can calculate curve data for any selected value date. To do this, first select value date, and then choose the type of the curve, finally Click “Calculate” button to get the result.

Value date can be any dates from the earliest maturity date to the latest maturity date that are displayed in the output table. Its format has the form of “dd-mmm-yyyy”

Click “Clear Contents” button to clear all the output data in the table. It’s not an obligatory step.

Notes for the yield curve output Section

Please make sure that you have curve data in the output table before plotting curves or calculating the curve data for any selected value date.

Tips for the MoneyMarket&Swaps Spreadsheet

There are convenient links on the right hand side of each orange band. By clicking “I”, “II”, “III” or “IV”, you can go to Section I, Section II , Section III or Section IV. Due to the large size of the spreadsheet, this feature is very helpful.

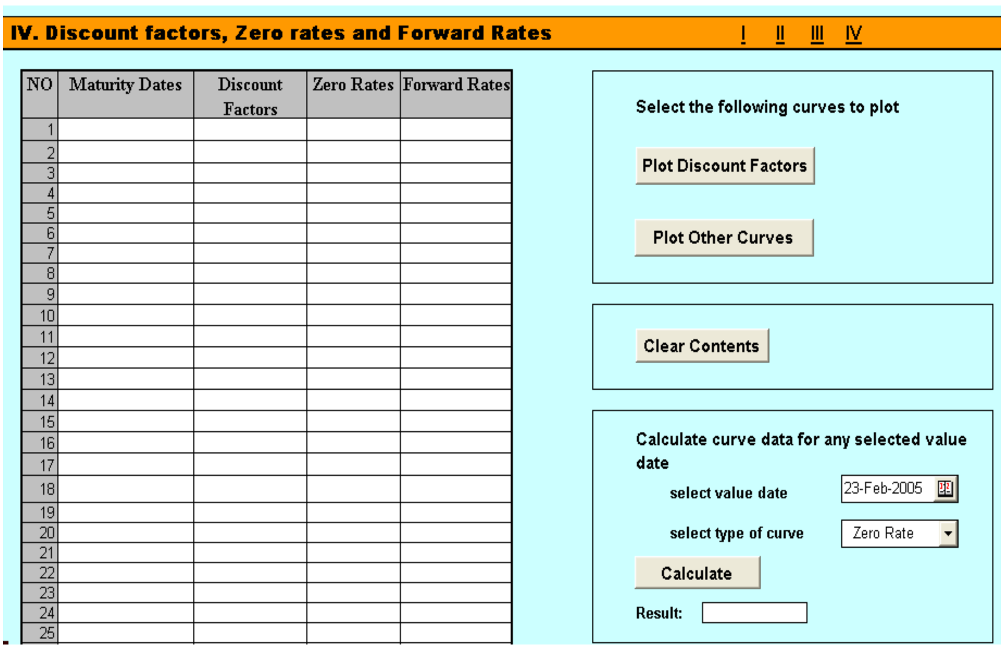


Fig 6

Chapter

2

Yield Curve Estimating and Fitting Methods

Basic Fixed Income Formulae

A. Discount Functions

If we have annual interest rate (spot or zero-coupon rate) *r*, the discount function with annual compounding can be calculated by

$$df = \frac{1}{(1+r)^B}$$

Where:
r is the zero rate;
df is the discount factor;
B is the Day Count Fraction, different day count basis will lead to different value of B and also different df .

B. Zero Rates

If we have the discount factor, the zero rate *r* with annual compounding can be calculated by

$$r = \frac{1}{df^{1/B}} - 1$$

where:

r is the zero rate;
df is the discount factor;
B is the Day Count Fraction, different day count basis will lead to different value of B and also different df .

C. Forward Rates

A set of forward rate periods are determined from value date to the last date of the curve using the frequency set in the Input Initial Parameter Section of the spreadsheet.

The forward rate is then calculated as follows:

$$F = \left(\frac{df_S}{df_E} - 1 \right) \times \frac{1}{B}$$

where:

df_S is the discount factor for the start of the forward rate period;
df_E is the discount factor for the end of the forward rate period;
B is the Day Count Fraction for the relevant forward period.

D. Day Count Fraction

Day Count Fraction B is determined by the following formula

$$B = \frac{d}{DaysOfYear}$$

where:

d is the number of days between the value date and the contract maturity, i.e. the period of the contract in days. For 30/360 day count basis, however, d does not equal exactly the actual number of days in the contract period since it's assumed each month has 30 days.
DaysOfYear is number of the days in a year. For Actual/Actual day count basis, it is the number of actual days in that year; for Actual/365, it is 365; for Actual /360 and 30/360, it is 360.

Therefore, different choice on day count basis will affect the value of day count fraction.

Curve Fitting Methods

A. Natural Cubic Spline

A Natural cubic spline is a smooth spline constructed of piecewise third-order polynomials which pass through a set of m control points. Natural cubic spline is an interpolation curve and can be used to connect the control points with smooth curve.

Consider a spline for a set of n+1 control points $(t_0, t_1, ..., t_n)$. Following Bartels et al. (1998), the ith piece of the spline can be represented by

$$Y_i(t) = a_i + b_i(t - t_i) + c(t - t_i)^2 + d(t - t_i)^3, i = 0,1,..., n-1;$$

and the spline curve as a whole can be expressed as

$$Y(t) = \begin{cases} Y_0(t) & t \in [t_0, t_1] \\ Y_1(t) & t \in [t_1, t_2] \\ Y_2(t) & t \in [t_2, t_3] \\ ... & \\ Y_{n-1}(t) & t \in [t_{n-1}, t_n] \end{cases}$$

So there is a total of 4n coefficients, $\{a_i, b_i, c_i, d_i\}_{i=0,...n-1}$ to be determined and we need at least 4n functions to constrain those coefficients. At each interior point, we have four conditions: curve passing through the point must have a continuous first and second order derivative, and the curve must pass through the point. This gives us four times the n-1 interior points = 4n-4 equations. We get two extra equations from the curve touching the exterior points, yielding 4n-2. The last two we choose artificially, such as setting the second derivative of the curve at the exterior points as 0.

B. Cubic B-Spline

The B-Spline is approximating spline curve, so it need not necessarily pass through every control point. It has advantages - such as the degree of the polynomial is independent from the number of points. Also, the shape is controlled locally, so adjusting a single point does not require total re-computation of the curve. It is a common practice to use B-splines to construct cubic splines for its favourable properties. The Cubic B-spline model discussed later uses B-splines to construct the smooth "discount function".

A B-spline is a generalization of the Bezier curve. Let a vector known as the knot vector be defined

$$T = \{t_0, t_1, ..., t_m\},$$

where T is a non-decreasing sequence.

Define the B-spline basis functions of degree *p* as

$$B_{k,0}(t) = \begin{cases} 1 \; if \; t_k \le t < t_{k+1} \; and \; t_k < t_{k+1} \\ 0 \quad otherwise \end{cases}$$

$$B_{k,p}(t) = \frac{t - t_k}{t_{k+p} - t_k} B_{k,p-1}(t) + \frac{t_{k+p+1} - t}{t_{k+p+1} - t_{k+1}} B_{k+1,p-1}(t)$$

Given S control points $(Q_i)_{i=0,\ldots,S-1}$, a B-spline curve of degree p is defined as

$$Y(t)=\sum_{k=0}^{S-1} Q_k B_{k,p}(t)$$

with respect to the knot sequence $t_0 \le t_1 \ldots \le t_{S+p-1} < t_{S+p}$.

The cubic B-spline curve is B-spline curve of degree 3 (i.e. p=3) and can be constructed using the equation above.

Discount Factors Estimation

YCF uses both Cubic B-Spline and Bootstrapping methods to estimate discount factors. The inputs for the former method are coupon bond data and the inputs for the latter are interest-rate swap rates data.

A. Cubic B-Spline Method

This method assumes the discount function as a smooth curve and constructs it using the Cubic B-Spline function with the following form,

$$df(x)=\sum_{k=0}^{S-1} Q_k B_{k,3}(x)$$

Where:

$df(x)$: is the discount factor w.r.t. date x;

Q : is the S-dimensional control vector whose components $(Q_i)_{i=0,\ldots,S-1}$ are the unknown control points of the Cubic B-spline curve.

$B_{k,3}(x)$: is the cubic B-spline basis function explained incurve fitting method chapter .

The theoretical value of a coupon bond can be expressed as follows:

$$\hat{P}_i=\sum_{j}^{ni} C_j^i df(x_j^i)=\sum_{j}^{ni} C_j^i \sum_{k=0}^{S-1} Q_k B_{k,3}(x_j^i)$$

where:

$(C_j^i)_{j=1,2,\ldots ni-1}$ is the coupon payment before the maturity date, and C_{ni}^i is the sum of the coupon and principal payment on the maturity date for the bond i;

x_j^i is the date at which C_j^i is expected to be paid.

The unknown coefficients of the discount function (vector Q) can be determined through minimizing the following function:

$$F(Q)=\sum_{i=1}^{N}\left(P_i-\hat{P}_i\right)^2+\lambda\int_0^T [df''(x)]^2\,dx$$

$$=\sum_{i=1}^{N}\left(P_i-\sum_{j}^{ni}C_j^i\sum_{k=0}^{S-1}Q_k B_{k,3}(x_j^i)\right)^2+\lambda\int_0^T\left(\partial^2\sum_{k=0}^{S-1}Q_k B_{k,3}(x)/\partial x^2\right)^2 dx$$

where:

P_i: is the observed market value(dirty price) of bond number i;

λ: is a parameter that determines the relative importance of goodness versus smoothness of fit to the observed data.

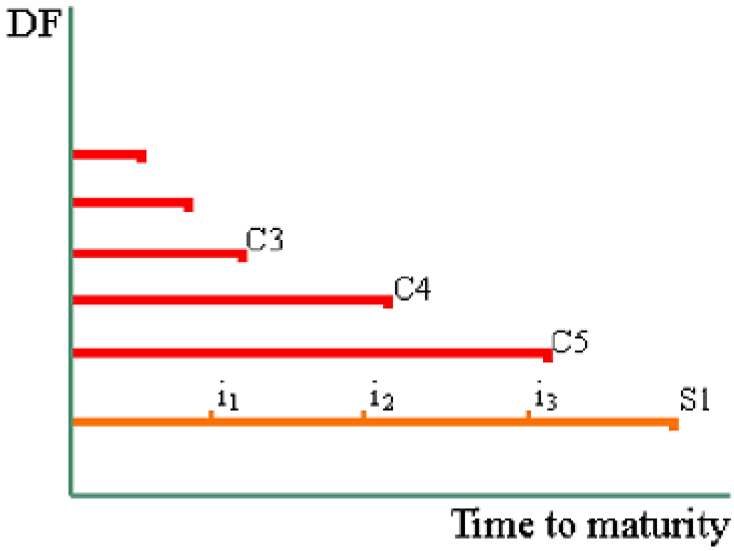
The first term measures the gap between a set of bonds' theoretical prices and their observed market prices, while the second term measures the smoothness of the discount function. By solving the minimization problem above, we can find the values for vector Q, through which one can construct a smooth and accurate discount function.

The knot points $\{t_0, t_1, ..., t_m\}$ of the Cubic B-Spline curve are connected to the maturity of the bonds used in the calculation.

B. Bootstrapping Method

This method calculates the discount factors in a bootstrapping fashion from successive swap rates combined with money market data. The quoted swap rates can be considered as par yields for bonds in the same class. So for a swap, its coupon equals its rates.

In the diagram below we show Discount Factors C3, C4, C5 that have been calculated from the money market rates. These, with some form of interpolation, are used to calculate an appropriate df (discount factor) for coupon points i1, i2, and i3.



The basic function to calculate the unknown df, i.e. the df for point S1 is:

$$df = \frac{N - (1 \times r \times \sum_{i=1}^{t-1}(df_i \times B_i))}{1 + (1 \times r \times B_t)}$$

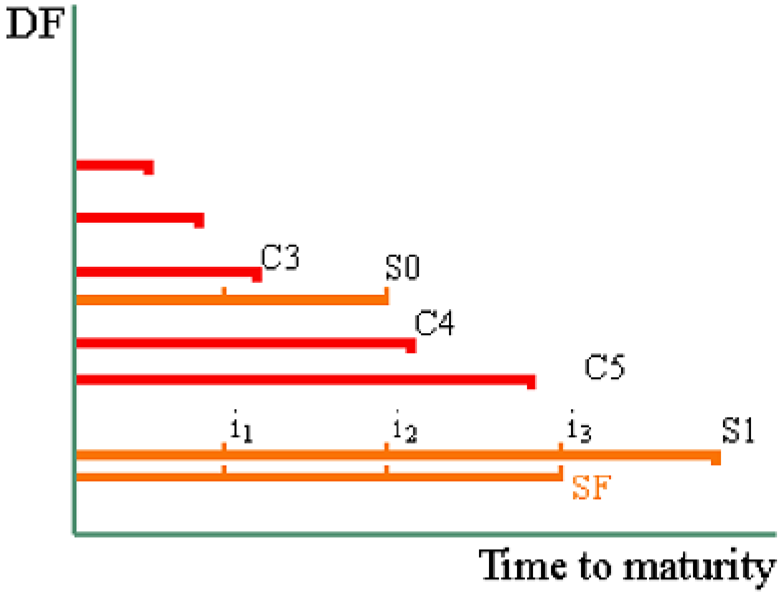
where:

N is Notional (or 1);
r is the quoted swap rate
df_i Discount factor for each coupon from 1 to t-1 (i.e. all excluding last coupon);
Bi Day Count Fraction for each coupon from 1 to t-1 (i.e. all excluding last coupon); and
t is the last coupon date.

In other words we can calculate the discount factor, df, for the period of the swap if we can extract the df_i for each and every coupon payment.

However, a small difficulty arises when the swap has a coupon, other than the last coupon, which falls beyond the previously calculated df.

In the diagram below coupon i3 falls after the last calculated df at point C5. We don't know the value of df for i3.



In order to solve this problem we create a fictitious swap. This fictitious instrument (SF) matches S1 in its coupon payment dates up to and including coupon i3 but excludes the last coupon. In other words the principal is repaid at point i3.

A discount factor for SF can now be calculated using the formula set out above. However, we do not have a swap rate to use. A rate for SF is manufactured by linearly interpolating between the previous swap rate (S0) and the next swap rate (S1), that is,

$$SF = S0 + \left[\left(\frac{S1 - S0}{T1 - T0} \right) \times (T - T0) \right].$$

where:

T: time to maturity for SF;
T0: time to maturity for S0;
T1: time to maturity for S1.

For this reason it is an important requirement that the yield curve market quotes for the cash overlap the first swap quote.

Each swap df can now be calculated in turn. S1 once calculated can be fed into the calculation of S2 and S2 into S3 and so on.

This is what is termed "bootstrapping". It is derived from the idea of doing up your bootstraps. You must start from the bottom and work your way up to the top. The next step is dependent on the previous step (hence an error early on will be magnified all the way up each subsequent step).

Yield Curve Construction

A. With Coupon Bond Data

If you want to construct a yield curve (e.g. zero-coupon yield curve, forward rate yield curve, etc.) using coupon bond data, please go to the Bond Market Page.

In the bond market page, the program first calculates discount factors using the Cubic B-Spline method. Then the zero-coupon rates and forward rates are calculated from the estimated discount factors. Further, the yields to maturity of a bond are interpolated using natural cubic spline interpolation.

To reduce the oscillations in the forward rate curve, we impose further smoothness restrictions on the calculated forward rates and get a smoother forward curve. The forward rate of the last period is assumed to be the same as that of previous period since there is no further discount data to calculate it.

B. With Money Market Data

If you want to construct a yield curve (e.g. zero-coupon yield curve, forward rate yield curve, etc.) using Money market data only, please go to the Money Market& Swaps Page.

In this page, please input money market rates and click the "calculate curve" button to get the results. The program first uses natural cubic spline to interpolate the money market rates, and then calculates discount factors from the zero rates. Forward rates are calculated from the estimated discount factors.

The forward rate of the last period is assumed to be the same as that of the previous period since there is no further discount data to calculate it.

You are recommended to choose "weekly" or "monthly" frequency for the Results/forward rates frequency input, following market practice.

C. With Money Market and Swaps Data

If you want to construct a yield curve (e.g. zero coupon yield curve, forward rate yield curve, etc.) using Money market data and swaps, please go to the Money Market& Swaps Page.

In this page, please input money market rates and swap rates, then click the "calculate curve" button to get the results. The program first uses natural cubic spline to interpolate the money market rates, and then calculates discount factors from the zero rates. The discount factors with maturities longer than the last money market rate are derived using bootstrapping method with swap data. Finally, forward rates are calculated from the estimated discount factors.

The forward rate of the last period is assumed to be the same as that of previous period since there's no further discounts data to calculate it.

Remember to have an overlap in maturity date between the last money market rate and the first swap rate.

Chapter

3

Troubleshooting and Q&A

Troubleshooting

This section provides a table showing errors you may encounter using YCF 2.0, probable causes for these errors, and suggested solutions.

Table 2: YCF 2.0 Errors and Suggested Solutions

Message	Probable Cause	Suggested Solution
Error in YieldCurveFitting.YieldCurveFitting.2_0: Error getting data conversion flags.	Usually caused by mwcomutil.dll not being registered.	Find the EXE file YieldCurveFitting.exe in the following directory: C:\WINDOWS\system32 or C:\WINNT\system32, and execute it.
Error in YieldCurveFitting.YieldCurveFitting.2_0: Undefined function or variables	Some required data have not been input in the spreadsheet.	Input all the data required.
Error in VBAProject: Automation error The specified module could not be found.	This usually occurs if MATLAB runtime lib is not on the system path.	Place %SystemRoot%\system32\bin\win32 on your system path.
Error in VBAProject: ActiveX component can't create object.	1. Project DLL is not registered. 2. An incompatible MATLAB DLL exists somewhere on the system path.	Find the EXE file YieldCurveFitting.exe in the following directory: C:\WINDOWS\system32 or C:\WINNT\system32, and execute it.

Table 2: Excel Errors and Suggested Solutions

Message	Probable Cause	Suggested Solution
Macros in this project are disabled. Please refer to the online help or documentation of the host application to determine how to enable macros.	The macro security for Excel is set to High.	Set Excel macro security to Medium on the Security Level tab (Tools > Macro > Security).
The Maturity dates in the result table are 4 years earlier than the true date	Excel is using 1904 date system instead of default 1900 date system.	To change to the 1990 date system, click Options on the Tools menu, click the Calculation tab, and then unselect the 1904 date system check box.

Frequently asked questions

- **How to install YCF 2.0**

Double click the Setup file in the installation CD, and install it according to the screen instructions. You must restart your computer before using this program.

During the installation, a command line window will pop up and ask you where to install the MATLAB runtime libraries. You should choose the default directory (just press ENTER to choose it), i.e. C:\Windows\System32 or C:\WINNT\System32, to install them. The Yield Fitting installation wizard will automatically add the installed directory to your PATH environment variable, and you don't have to do it manually.

If MATLAB runtime libraries have been previously installed in the other directory rather than C:\Windows\System32 or C:\WINNT\System32, you must un-install them before you continue this Installation Wizard.

- **How to access YCF 2.0**

You can start YCF 2.0 by clicking the YCF2.0 icon in the desktop or from Start Menu/Programs/YieldCurveFitting2.0.

- **How to remove YCF 2.0**

To remove YCF 2.0, please go to Settings/Control Panel/Add &Remove programs, Select it and click the "Remove" button.

- **Can I save YCF2.0 spreadsheet file**

YCF 2.0 is a Read-Only spreadsheet file, and you cannot save the modified YCF2.0 with its original name. However, you can always save it using a different name. Though you can construct the yield curve with your saved new spreadsheet, errors may occur due to the changes of the structure in the spreadsheet.

References:

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