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A SPECIAL REVIEW BY MOHAMOUD DUALEH AND ABUKAR ALI
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Standardised INTEREST-RATE SWAPS:

assessing the LIFFE Swapnote®

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The ubiquitous interest-rate swap has proved itself, over some not inconsiderable time, to be one of the great success stories of financial derivatives. It is arguably the most widely-traded derivative contract and has lent itself to constant development and application. Its flexibility derives in part from its over-the-counter (OTC) bespoke nature. Paradoxically, financial institutions have also considered that an exchange-traded swap contract, transacted along similar lines to exchange-traded futures, may present unique advantages in combining some of the flexibility of the OTC swap with the usefulness of standardised contracts.

Thus financial institutions have in recent years considered a new class of interest-rate futures products – the exchange-traded swap contract. On March 20, 2001 LIFFE launched Swapnote®. The Swapnote® contract is essentially a forward starting swap contract that cash settles on the start/effective date of the underlying swap. Following its introduction, trading volume in this instrument can now be considered liquid. The success of the Swapnote® is due to its simplicity in creating a standardised exchange-traded futures contract but combining the price sensitivity of an interest-rate swap. As such it can be used for a variety of hedging and trading purposes.

In this article we explore some methods of evaluating Swapnote® futures. First, we consider briefly the take-up of Swapnote®. We look at evaluating Swapnote® using one of the standard interest rate models. We then consider briefly the standard method for computing the convexity adjustment necessary when using the contract for cash market hedging, before concluding the article.

SWAPS AND A NEW BENCHMARK

The Swapnote® has introduced a new dimension to the interest-rate swap market. Swapnote® is a family of futures contracts that allows institutions to access the euro interest-rate swaps market. It is dependent on the euro swap yield curve, usually simply called the swap curve. In terms of notional volume the swap market is the largest fixed income market in the world, being approximately six times the size of the bond market.[1]

In an era of diminishing liquidity in government bond markets, and the advent of the euro, the swap market has become the benchmark for long-term interest rates. Hence the swap curve has become the primary means of price discovery in the euro-denominated fixed income market. The factors which contributed to this included the size and homogeneity of the swap market, compared to combined government bond markets, and the decline in government bond issuance, together with growing non-government and corporate bond issuance.[2] The turnover of the swap market on the other hand, consistently trends upwards, for instance it grew by 104% between 1998 and 2001.[3]

From a trader's point of view, the best hedges are achieved through vehicles that are highly liquid and as closely correlated with the underlying assets as possible. In Europe (and arguably in other markets) the interest rate swaps market now appears to achieving this better than the government debt markets. The efficacy of the swap curve as a benchmark reflects the fact that the euro swaps market is now larger than the Eurozone government bond market.

Exchange-traded swap contracts are traded on the London International Financial Futures and Options Exchange (LIFFE) and the Chicago Board of Trade (CBOT). Both are based on the future value of a swap rate. The Swapnote® futures contracts have grown rapidly since their introduction in March 2001. For traders and investors with exposure to credit risk based on Libor, the Swapnote® contract has been accepted as an effective hedging tool as, for instance, the US Treasury futures contract does not take credit risk into account. Hedging a swap portfolio with government bonds appears to work extremely well, however the assumption that the bond-swap spread remains constant is, in practice, not realistic. The spread often exhibits significant volatility.[4] In fact, hedging with government bond futures presents significant *basis risk*. The Swapnote® is designed to both strengthen the benchmark status of the swap curve and provide an effective and accessible hedge for a portfolio of securities. It should also reduce basis risk for practitioners looking to hedge (say) corporate bond portfolios.

THE SWAPNOTE® CONTRACT

For readers reference we provide first an overview of the main features of the Swapnote® contract. Swapnote® is essentially a forward starting swap contract that cash settles on the start or effective date of the underlying swap. The contract can also be regarded as a cash-settled bond futures contract with a single notional bond in the deliverable basket. The contract is essentially similar to a standardised exchange-traded futures contract but with the price sensitivity of an interest rate swap. Each Swapnote® contract has a series of notional cash-flows underlying it comprising a fixed coupon element together with a principal repayment such that it replicates a notional bond.

At first, the coupon level is set at 6% for each of the contracts, thereby facilitating spread trading between government bond futures contracts and Swapnote® contracts of related maturity. Similarly the contracts are timed on a quarterly expiry cycle for the months of March, June, September and December, the usual expiry months for futures exchanges, and denoted by the letters H, M, U and Z. Exhibit 1 illustrates the 10-year Swapnote® contract specification.

Swapnote® is a standardised exchange-traded futures contract. In other words, the 10-year swap futures will all be in the same contract month. With Swapnote®, if we assume that there is sufficient liquidity[5] it may be easier to execute a buy or sale of 10-year swap futures than it currently is to buy or sell a 10-year strip of Eurodollar futures contracts.

We show trading volumes for the Swapnote® futures contract in Exhibit 2. During 2003 volumes experienced new highs, with the 10-year contract recording a daily volume high of 55,261 in June 2003 (June 13, 2003) and the five-year contract setting a new volume high record of 57,761 in the same month.[6]

10-year Swapnote® - contract specification	Exhibit 1
€100,000 notional principal amount	Last trading day 11:00 Brussels time (10:00 London time) Two London business days prior to the delivery day
Notional coupon 6.0%	
Maturities 2 Notional principal amount due 10 years from the delivery day	Quotation Per €100 nominal value
Delivery months March, June, September and December such that the nearest two delivery months are always available for trading	Minimum price movement (Tick size and value) 0.01 (€10)
Delivery day Third Wednesday of the delivery month	Trading hours on LIFFE CONNECT™ 07:00 - 18:00

Source: LIFFE

EVALUATING THE SWAPNOTE® CONTRACT

The price of the Swapnote® contract until its expiration date will reflect underlying supply and demand risk factors. At expiration, the contract settles based on the Exchange Delivery Settlement Price (EDSP) fixed by the exchange. The EDSP is defined as the sum of the discounted notional cash-flows, each of which has been present valued using zero coupon discount factors derived from the ISDA Benchmark Swap Rates on the last trading day. The discount factors are zero coupon rates bootstrapped from the current par swap curve. Cash-flow payment dates are defined as anniversary dates of the effective date. However, should any of these dates fall on weekend or holiday, notional cash-flows are moved to a business day.

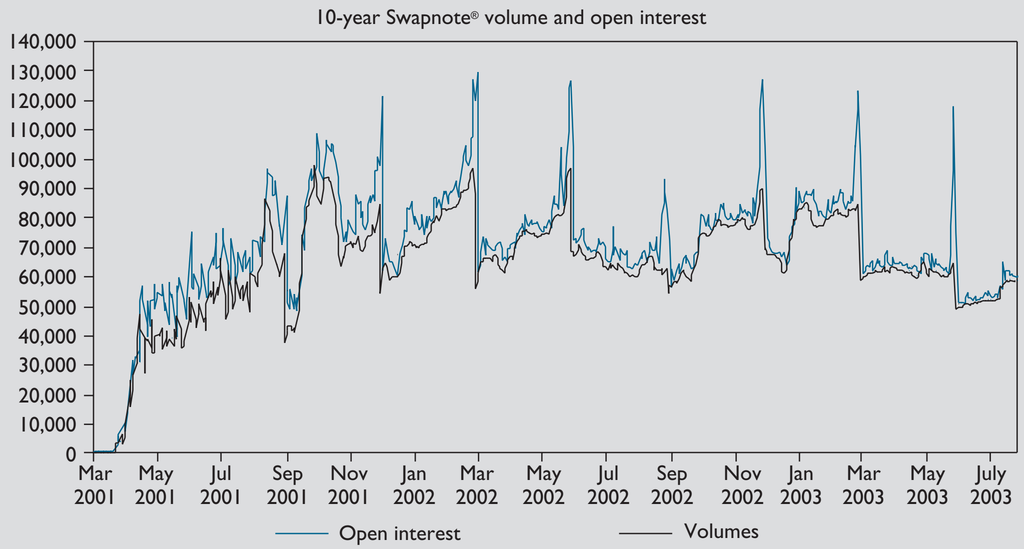
Users of the Bloomberg system can use the Bloomberg Swapnote® Futures Analysis function, screen FVD, to evaluate the fair value of a Swapnote® contract, which is illustrated in Exhibit 3. It is obtained by typing

P A Cmdty FVD <GO>.

Screen FVD (see Exhibit 3) shows the market value of the Swapnote® and its conventions such as the day count and valuation date. The Swapnote® can be priced as a forward starting swap where the swaps effective date is set as the valuation date of the futures contract. The sensitivity measures from the FVD screen can be replicated by pricing a 10-year euro-denominated bond with a forward settlement date of the futures valuation date and maturity date, day count and frequency from the futures contract. Note from Exhibit 3 that a 6% notional coupon is used as the bond's fixed coupon rate. In our example we have evaluated the 10-year Swapnote®. Exhibit 4 is page 2 from the same screen, and lists the fixed coupon and forward rates at each interest fixing date. The forward rates as at

Swapnote® trading volumes

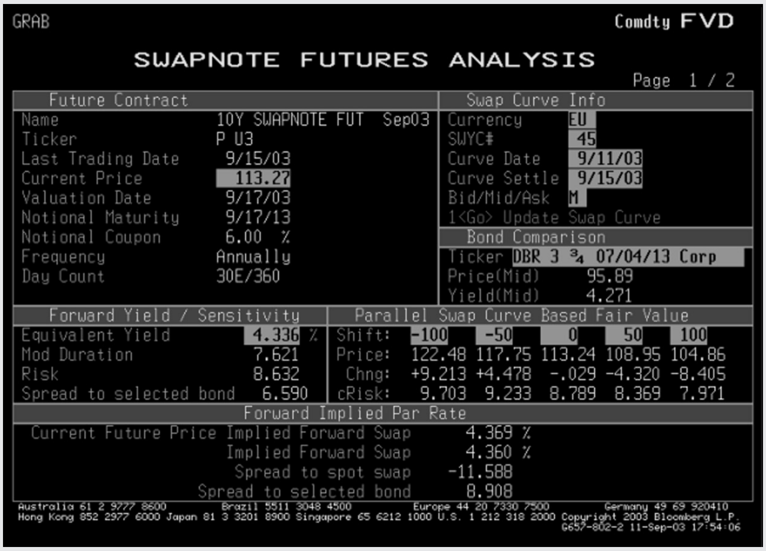
Exhibit 2



Source: LIFFE. Used with permission.

Bloomberg screen FVD for 10-year Swapnote® contract, as at September 11, 2003

Exhibit 3



Source: Bloomberg L.P. Used with permission.

each fixing date are also shown. Screen FVD assesses Swapnote® against a comparison bond. This defaults to the current 10-year German government bond, shown to be the 3.75% 2013 bond. The 'equivalent yield' shown is the notional yield to maturity of a government bond with a 6% yield priced to settle on the valuation date, and maturing exactly 10 years from the valuation date. The spread to the comparison bond is shown to be 6.59 basis points.

The Swapnote® delivery method is cash settlement. The final settlement value will be determined as

$$X * [6/r + (1 - 6/r)*(1 + 0.01*r/2)^{-20}]. \quad (1)$$

The underlying notional cash-flows consist of a series of fixed notional coupons and a notional principal at maturity, the dates of which fall on anniversaries of the delivery day. Once the cash-flows from futures contract are implicit, any of the standard interest rate models can be used to price Swapnote® contracts.[7]

Of course it is not essential that a proper term-structure model should be used. Fundamentally, the Swapnote® quoted price corresponds to the forward value of a bond having a 6% coupon. At maturity, the settlement price is computed by discounting the 6% coupons plus principal using a zero coupon curve derived from the official swap rate fixings.

To illustrate, using linear interpolation we priced the two-year, five-year and 10-year Swapnote®s. First we constructed a zero curve off deposit and swap rates and discounted the coupons and the notional principal of the underlying swap. From Exhibit 5 we show the quoted prices and our theoretical prices. Note that we converted those into equivalent bond yields to get a difference in term of basis points. In fact we can do the same analysis using Bloomberg.

We reiterate that the Swapnote® contract is essentially a forward starting swap contract that cash settles on the start/effective date of the underlying swap. Thus, a swap

Bloomberg screen FVD, page 2 — **Exhibit 4**

GRAB — Comdty FVD

SWAPNOTE EXPIRY DAY ANALYSIS

Page 2 / 2

Notional Value Dates	Notional Coupon amounts	Curve Derived Disc. Factors	Forward Swap Rates	Expiry Day Settle Price
				113.23
9/17/03		0.99988424		
9/17/04	6.00000000 %	0.97710051	2.332 %	
9/19/05	6.03333333 %	0.94657311	2.764 %	
9/18/06	5.98333333 %	0.91087056	3.137 %	
9/17/07	5.98333333 %	0.87262840	3.432 %	
9/17/08	6.00000000 %	0.83352702	3.663 %	
9/17/09	6.00000000 %	0.79426123	3.854 %	
9/17/10	6.00000000 %	0.75516532	4.018 %	
9/19/11	6.03333333 %	0.71682427	4.156 %	
9/17/12	5.96666667 %	0.68041108	4.267 %	
9/17/13	6.00000000 %	0.64520653	4.361 %	

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Source: Bloomberg L.P. Used with permission.

Comparison of quoted and theoretical prices — **Exhibit 5**

	2-year	5-year	10-year
Quoted	106.06750	110.46000	113.52500
Theoretical	106.07138	110.46619	113.51066

Source: YieldCurve.com

position can be interpreted as a package of forward/futures contracts. Future contracts are designed to remove the risk of default[8] inherent in forward contracts. Through the device of *marking to market*[9], the value of the future contract is maintained at zero at all times. Thus, either party can close out his/her position at any time. This difference gives rise to the *convexity bias*. When hedging or pricing across futures and swaps markets, the issue of the convexity bias will lead to hedge risk.

If one is pricing the Swapnote® as a forward starting swap using the swap curve, then there is no issue: the swap curve already exhibits a convex feature. Hence there is no need to adjust for the convexity effect. If however one wishes to price the swap off the Euribor futures contracts, then one will need to adjust the curve defaults for convexity bias. The Bloomberg Swapnote® Futures calculator does not adjust for convexity because it uses the market swap curve (see Exhibit 3) and not the curve derived from futures contracts to calculate the implied forward rates needed when pricing swaps. Therefore, when using the Euribor futures, the back month futures do not reflect the true Euribor forward rates and the volatility of rates required for valuing swaps. To overcome the imperfections in hedging that this will cause, one must adjust for this bias. The simplest approach is to use one of the standard interest rate models but with a convexity adjustment.

In Exhibits 5 and 6 we are comparing the Swapnote® settlement prices with those of the Euribor futures which settle at around the same time. The main difference between these methods of evaluating the Swapnote® is that the last method takes into account the convexity correction discussed below. Exhibit 6 illustrates pricing Swapnote® off the Eurodollar futures market using the LIFFE Swapnote® calculator, together with a convexity correction using the Kirikos & Novak equation. We look at this issue in greater detail later.

PRICING FRAMEWORK

We can now formulate the framework that leads to the standard methodology for pricing the Swapnote® contract. Let us start first by defining the accumulation factor (i.e. the saving account) as

$$\beta(t) = \exp\left[\int_0^t r(u)du\right] \qquad (2)$$

A zero coupon bond, maturing at time T, pays US$1 at time T and nothing before time T[10]. Intuitively, Equation (2) represents the price process of a risk-free security which continuously compounds in value at the rate r. We first consider the situation with discrete trading dates

Pricing Swapnote® off of the Eurodollar futures market			Exhibit 6
	2-year	5-year	10-year
Uncorrected	105.697	107.094	104.386
Theoretical	105.702	107.505	106.273
Settle	**105.720**	**107.52**	**106.26**

Source: YieldCurve.com

$$0 = t_0 < t_1 < \dots < t_n = T$$

On each $[t_j, t_{j+1})$, r is constant, so

$$\beta(t_{k+1}) = \exp\left\{\int_0^{t_{k+1}} r(u)du\right\} = \exp\left\{\sum_{j=0}^{k} r(t_j)(t_{j+1} - t_j\right\} \tag{3}$$

is $I(t_k)$ -measurable[11].

Suppose we enter a future contract at time t_k, taking the long position, when the future price is $\pi(t_k)$. At time t_{k+1}, when the future price is $\pi(t_{k+1})$, we receive a payment of $\pi(t_{k+1}) - \pi(t_k)$. We pay $- \pi(t_{k+1}) - \pi(t_k)$ if the future price has fallen. The process of paying and receiving these payments is the margin account held by the broker.[12]

By time $T = t_n$, we will have received the sequences of payments

$$\pi(t_{k+1}) - \pi(t_k),\ \pi(t_{k+2}) - \pi(t_{k+1}),\dots,\ \pi(t_n) - \pi(t_{n-1})$$

at times $t_{k+1}, t_{k+2},\dots,t_n$. The value of this sequence is

$$\beta(t)\mathrm{E}\left[\sum \frac{1}{\beta(t_{t+1})}(\pi(t_{j+1}) - \pi(t_j)\Big| I(t)\right]. \tag{4}$$

The futures price for a contract is defined as the delivery price that makes the contract have zero value at the time that the contract is entered into. Because it costs nothing to enter the future contract at time t, the above expression must be zero almost surely.

Cash-flows

As we mentioned earlier, once the cash-flows from Swapnote® futures contract are known, any of the standard interest rate models can be used. With a future contract, entered at time 0, the buyer receives a cash-flow between times 0 and T. If we hold the contract at time T, then we pay $V(T)$ at time T for an asset valued at $V(T)$. Thus, the cash flow received between times 0 and T sums to

$$\int_0^T d\pi(u) = \pi(T) - \pi(0) = V(T) - \pi(0). \tag{5}$$

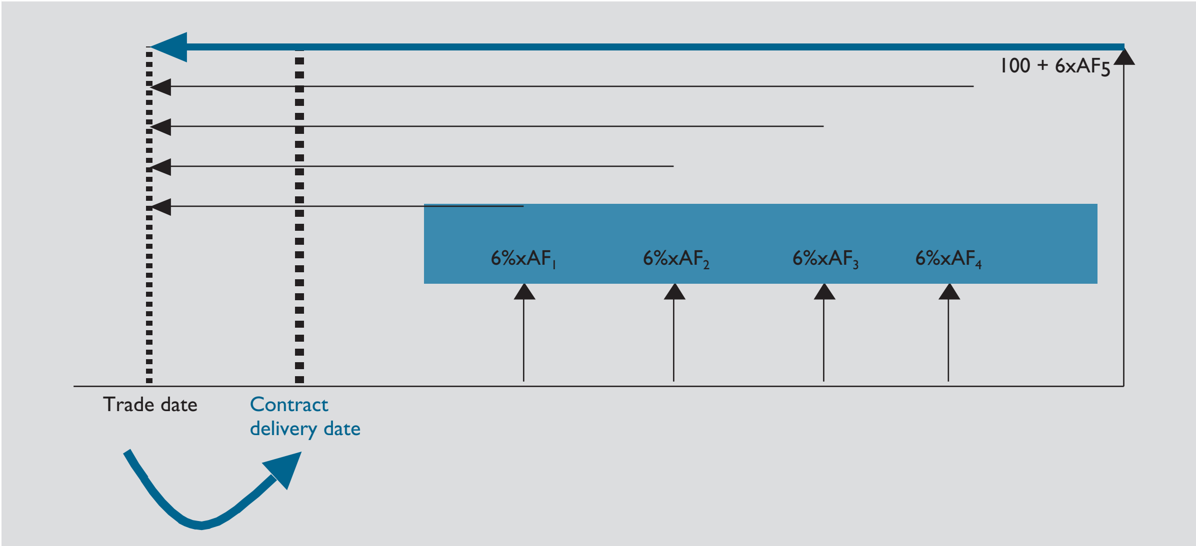
Therefore, if we take delivery at time T, we paid a total of

$$(\pi(0) - V(T)) + V(T) = \pi(0)$$

for an asset valued at $V(T)$.

Exhibit 7 illustrates the arbitrage-free pricing of a five-year, 6% Swapnote®. The notional cash-flows are present valued to the contract trade date, summed and financed to delivery.

Arbitrage-free pricing of a five-year, 6% Swapnote® **Exhibit 7**



Source: YieldCurve.com

Forward - future spread

Earlier we suggested an interpretation of a swap as a package of forward/futures contracts, given that we can now look at the forward/futures spread.[13] First let us define the future price and the forward price.[14]

Future price: $\pi(t) = E\left[V(T)|I(t)\right]$.

Forward price: $F(t) = \dfrac{V(t)}{B(t,T)} = \dfrac{V(t)}{\beta(t)E\left[\frac{1}{\beta(T)}|I(t)\right]}$

We can derive the difference between the forward bond price by using a zero-coupon curve and the future bond price.

Let $E\left(\frac{1}{\beta(T)}\right)$ be the discount factor until time T.[15]

The forward – future spread is

$$\pi(0) - F(0) = E\left[V(T)\right] - \frac{V(0)}{E\left[\frac{1}{\beta(T)}\right]}$$

$$= \frac{1}{E\left(\frac{1}{\beta(T)}\right)}\left[E\left(\frac{1}{\beta(T)}\right)E(V(T)) - E\left(\frac{V(T)}{\beta(T)}\right)\right] \qquad (6)$$

If $\frac{1}{\beta(T)}$ and $V(T)$ are uncorrelated, then $\pi(0) - F(0)$.

Assuming that $V(T)$ and $\frac{1}{\beta(T)}$ are perfectly correlated, then we have, using last expression of Equation (6), and rearranging

$$E\left(\frac{V(T)}{\beta(T)}\right) = E\left(\frac{1}{\beta(T)}\right) E(VT) + \sigma V(T)^{*}\sigma\left(\frac{1}{\beta(T)}\right) \qquad (7)$$

Therefore, the forward –future spread is given by

$$\textit{Forward - Futures Spread} = \left(\frac{1}{\beta(T)}\right)^{*}\sigma V(T)^{*}\sigma\left(\frac{1}{\beta(T)}\right) \qquad (8)$$

$\beta(T)$ denotes the saving account value at time T (for US\$1 invested at time 0) and $V(T)$ the value of the asset considered at time T.

To calculate the standard deviation of $\beta(T)$ and $V(T)$ it is best to use the one factor Hull-White model because other standard interest rate models such as Black Karasinski, Black-Derman-Toy and Cox-Ingerosll-Ross are more knotty.[16] Basically the core difference between the models is skew, and the convexity correction is mainly an at-the-money phenomenon which is not very sensitive to skew.[17] Thus, the H&W model is less opaque and compute intensive. In the next section we consider the convexity correction.

Convexity adjustment estimation

A tailed hedge for the money market swap possesses a very desirable property, namely the net value of the swap plus hedge is positive irrespective of rates going up or down. Earlier we derived the forward/futures spread to be zero. However, if the futures position is greater (i.e. gaining) than the forward position, then as compensation, there must be an adjustment. There are several approaches of measuring the convexity effect (i.e. the convexity bias in futures)[18], but the Kirikos and Novak equation using Hull and White is a robust treatment for the convexity correction. For a rate that applies between time t and time T, under the Hull and White model the difference between the forward and futures rates is expressed as the z parameter of the popular Kirikos & Novak equation.[19]

Convexity adjustments for several futures markets are provided by brokers or from market data vendors. Estimating the convexity adjustment requires an estimation of the future path of interest rates up to the future contract maturity. In the Hull-White model, the continuously compounded forward rate, lasting between times t and T (denominated in years from current date), equals the continuously compounded future rate with the following adjustment factor e^Z. Of course this is the Kirikos & Novak factor,

where $Z = \Lambda + \Phi$

$\Lambda = \sigma^2 \left(\frac{1 - e^{-2at}}{2a}\right)\left[\frac{1 - e^{-a(T-t)}}{a}\right]^2$ adjusts for the fact that the underlying is an interest rate, and

$$\Phi = \frac{\sigma^2}{2}\left(\frac{1 - e^{-a(T-t)}}{a}\right)\left[\frac{1 - e^{-at}}{a}\right]^2$$

where σ is the standard deviation of the change in short-term interest rates expressed annually, and a is the mean reversion rate.

Convexity bias estimation requires an estimate of the mean reversion rate (a) and the standard deviation (σ) of the change in short-term interest rates expressed annually. For simplicity, LIFFE assumes a constant default value for the mean reversion speed. The LIFFE US$ Swapnote® calculator assumes that the mean reversion parameter (a) remains at 0.03. Bloomberg does allow convexity adjustment for pricing swaps if the underlying interest rate curve is based on traded futures contracts. There are two input parameters that are required in order to adjust futures contracts for convexity: the mean reversion speed which has a default level of 0.03 and volatility parameters which are fed from market traded implied cap/floor rates. There are various alternative methodologies for estimating the volatility parameters (σ and a). The two most popular methodologies are:

- to estimate the volatility parameters from prices of traded securities; and
- as discussed above, a deposit/swap derived curve could be used to estimate the volatility parameters.

CONCLUSION

Hedging a swap portfolio with government bond curves presents significant basis risk. A futures contract referenced to the swap curve provides a far more effective hedging tool with appreciably reduced basis risk. To address the problem of basis risk one can use a Swapnote® contract. The usefulness of Swapnote® as a hedging tool includes the ability to match and hedge credit exposures with a derivative instrument that closely correlates with that exposure. The capacity to hedge swap book exposures and avoidance of convexity, plus avoidance of problems that can be associated with government bond contracts under adverse market conditions, are key advantages.

The Swapnote® has allowed institutional investors to access the euro interest rate swaps market in standardised fashion. It is simple to price, once the cash flows from futures contracts are known. Any of the standard interest rate models can be used to evaluate the contract. The futures/forward spread can be adjusted using the Kirikos & Novak equation.

Finally we have shown how the convexity bias adjustment can be effected by following a straightforward approach and which will allow Swapnote® to be applied across futures and swaps markets for effective hedging.

Notes:

1. Source: ISDA, "Summary of OTC Derivative Market Data", www.isda.org/statistics.
2. A number of other factors have contributed to this, resulting in greater use of the swap curve to as the euro Benchmark. For a general discussion of government bond market illiquidity and the issues behind alternative benchmarks, see Choudhry (2003).
3. Source: www.isda.org/statistics. For a good discussion and related issues see Remolona and Wooldridge (2003).
4. See Flavell (2001), chapter 9 for more discussion about how the optimal hedge effectiveness in this regard can be rather low.
5. From anecdotal evidence the authors conclude that there is sufficient liquidity in the market. However given the relative youth of this instrument, this cannot be assumed to be permanent and continued observation would be prudent.
6. The authors express thanks to Kumud Chavda from LIFFE for providing these figures.
7. For instance, the Black 76, Cox-Ingersoll-Ross, Black-Derman-Toy and Hull-White models, to mention a few.
8. The default risk in a swap agreement is the counterparty risk.
9. That is, at end of each trading day, the margin account is adjusted to reflect the investors gain or loss.
10. Our approach follows that of Shreve (1997). There are number of sources on derivatives instruments that one can access on this issue, for instance Hull (2000). For mathematics of derivatives we refer the reader to Shreve (1997).
11. Some authors write I(t) as F(t), it is matter of choice!
12. A good reference about the margin requirement is Fabozzi (2003).
13. Basically we can view the swap spread as the futures/forward spread.
14. It is important to keep in mind that futures affect the swap through both estimation and discounting process.
15. Until expiry
16. There is no closed form solution for these models. You could use Monte Carlo simulation for example to calculate the payments along each path and discount them back.
17. Thanks to Patrick Hagan for pointing this out. See Hagan (2003).
18. See Richard Flavell, Swaps and Other Derivatives, Wiley (2000), pp 185-203.
19. See G. Kirikos and D. Novak (1997). This is also available at www.powerfinance.com/convexity

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