**Determining the expression for the fair value of the swap spread: an approach using Libor and GC repo rates**
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## Introduction

Several studies have highlighted an environment of lower liquidity in government bond markets, with yield levels falling below true risk-free rates.[1] Under these circumstances capital market practitioners are increasingly using the interest rate swap curve as their main reference benchmark for valuation and analysis purposes. This reflects practitioners' views that continued exclusive reliance on government yields would possibly lead to inaccurate analysis. The interest rate swap curve is viewed as an acceptable alternative measurement benchmark. It is also used as a comparator instrument, against which the yields on government bonds can be measured. As the interest rate swap market is considered to be sufficiently liquid and exposing users to only a very low level of default risk, the swap curve is considered invariably to be trading at a fair value to the theoretical zero-coupon yield curve. For this reason market participants are concerned that the swap curve is indeed fairly valued at all times, so that swap spreads over the government curve can be taken to be a reasonable measure of the liquidity and fair value of government bonds.

In this article we provide an expression for the fair value for the swap spread, which is shown to be a function of the Libor rate and the general collateral repo rate. This spread can then be used as an alternative benchmark measure used to assess relative value for all debt market instruments.


## I        Interest rate swaps

In setting fair value in the debt market, the only plausible alternative to the government yield curve in the euro and sterling market is the interest rate swap. A conventional interest rate swap is an over-the-counter derivative contract between two parties in which one party makes fixed interest payments, calculated on a notional amount, while the other party makes floating-rate interest payments. The fixed rate is set at the inception of the contract and the floating-rate is linked to an external reference such as Libor during the life of the swap.[2]

The fixed-rate payer in a swap is conceptually long a floating-rate bond (FRN) and short a fixed coupon bond, while the opposite is true for the floating-rate payer. When calculating the fair value swap rate we use the concept of the *par* swap, which is a vanilla swap of zero net present value. This means that the present value of the cash

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[1] For instance see Jeanneau and Scott (2001), McCauley and Remonola (2000) and Schinasi, Kramer and Todd Smith (2001).

[2] The typical sterling swap pays semi-annual interest. Only the net difference between the two interest amounts is exchanged. The notional or "principal" amount is not exchanged.

flows of the fixed and floating legs of the swap are equal. The fixed leg of the swap is identical to the interest payments of a bond with fixed coupon equal to the swap rate, while the floating leg is identical to the cash flows of an FRN with coupon set at the Libor rate. At the start of the contract the FRN and the fixed bond must be equal to the notional swap value. In a par swap therefore the fixed leg is conceptually similar to a fixed coupon bond priced at par, with a coupon equal to the swap rate. The swap rate in a par swap is therefore known as the *par yield*.

The observed difference between the swap rate and the government bond yield at any point on the yield curve is known as the *swap spread*. The swap spread is a measure of interbank risk, given that the government curve is assumed to be default-free and the swap market is used by banks and corporates to hedge their interest rate risk. Traditionally the swap spread was taken to be the measure of corporate default risk, with a corporate bond asset-swap spread taken as an indicator of the credit risk on that reference bond. However the swap market is now very large and liquid, and does not suffer from illiquidity out to the long-dated maturities. There are also no supply constraints in the swap market, unlike for (say) long-dated gilts or Treasuries, and the use of collateralisation, margining, netting and other measures has substantially reduced counterparty risk. Government bond markets on the other hand, have experienced low liquidity and supply constraints leading to inverted curves, leading some commentators to suggest that the government yields have traded below the true risk-free level; see Fleming (2000, 2001). In other words, the government curve may on occasion be overvalued, whereas the swap curve is now almost invariably at fair value.

The assumption that the swap curve is always at fair value, whereas the government curve at times may not be, is open to some debate. This is not least because the swap curve is a function of market makers bid-offer swap rates, and in the event of a significant shock or market correction the book position and supply of certain prices will influence the swap curve, affecting its use as a benchmark.[3] However if we make this assumption, we are then in a position to make an assessment of the swap spread and, by taking its fair value, of the government curve fair value as well. Assuming that the swap rate is at fair value, we can use the extent of the swap spread itself to determine if the government curve is at fair value; if the swap spread differs from that suggested by the swap rate fair value, this indicates a government curve that is overvalued.

In the next section we consider the expression for the fair value for the swap spread.


**II      Swap spread fair value**

We now consider the calculation of the fair value for the swap spread. This is done using a zero-cost portfolio trading approach, consisting of the benchmark government bond (paying fixed coupon on a semi-annual basis), the six-month government repo rate and the six-month Libor rate. We ignore bid-offer spreads.

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[3] Events such as the LTCM crisis or the emerging market fall outs of 1997 and 1998, for example.

A market participant enters into the following trades simultaneously:

- a short sale of € $x$ million of the benchmark []-year semi-annual paying government bond, which is priced at par and therefore trading at the par yield of $rm\%$; the bond is available in the repo market at the general collateral (GC) rate;

- a six-month reverse repo, using the cash proceeds from the bond sale; this pays out the six-month GC repo rate $rg\%$;

- a € $x$ million []-year pay floating / receive fixed interest-rate swap, where the trader receives six-month Libor or $rl$ and pays at the swap rate $rs\%$.

This sets up a zero-cost portfolio, which comprises the following cash flows:

- pay out of $rm$ every six months;

- receipt of $rg$ every six months;

- pay out of $rs$ every six months;

- receipt of $rl$ every six months.

The net cash flow of the combined portfolio during the time it is held is given by

$$Port = nom \times \left[ \left( \frac{rs - rm}{2} \right) - \left( \frac{rl - rg}{2} \right) \right] \tag{1}$$

where *nom* is the notional value equal to € $x$ million.

It can be shown that the present value of the portfolio cash flows during its life is given by

$$PV = \sum_{i=1}^{n} \frac{\frac{x}{2}(rs - rm)}{\left(1 + \frac{rz_i}{2}\right)^i} - E_0 \sum_{i=1}^{n} \frac{\frac{x}{2}(rl_i - rg_i)}{\left(1 + \frac{S_i}{2}\right)^i} \tag{2}$$

where $n$ is the maturity (in years) of the bond and swap and $x$ is the nominal value as before. The term $rz_i$ is the theoretical $i$-year risk-free spot rate interest rate.

The first term in the expression is the present value of the cash flows of the bond and the fixed leg of the swap. The second term is the present value of the *expected* difference between the floating rate of the swap and the repo rate. These are not known with certainty, as they will be set by the Libor fixing every six months and the repo rate as

this is rolled over at the same time and for the same term. As these cash flows cannot be determined at the inception of the trade, they are discounted at the rate of $s_i$, which is the spot rate $rz_i$ plus an additional element $t$, where $t$ represents the risk factor arising from the change in the spread $rl - rg$. The additional amount $t$ is user-determined. So we have

$$s_i = rz_i + t . \tag{3}$$

We assume that the swap rate curve and the benchmark government bond are fairly valued. This being so, the expression at (2) must be equal to zero, because the portfolio set up by the trader is a zero-cost transaction. This enables us to set

$$\sum_{i=1}^{n} \frac{\frac{x}{2}(rs - rm)}{\left(1 + \frac{rz_i}{2}\right)^i} = E_0 \sum_{i=1}^{n} \frac{\frac{x}{2}(rl_i - rg_i)}{\left(1 + \frac{s_i}{2}\right)^i} . \tag{4}$$

We can now set the swap spread in terms of the expected spread between the Libor and GC repo rates. This is done by re-arranging (4) and moving $(rs - rm)$ outside the summation, shown as

$$rs - rm = \frac{E_0 \sum_{i=1}^{n} \left[ (rl_i - rg_i) / \left(1 + \frac{s_i}{2}\right)^i \right]}{\sum_{i=1}^{n} \left[ 1 / \left(1 + \frac{rz_i}{2}\right)^i \right]} . \tag{5}$$

That is, the fair value of the swap spread is given by the expected spread between the Libor rate and the repo rate. By setting a risk factor for this spread $t$ as zero, the fair value of the swap spread is given by the weighted average of future expectations of the spread during the term of the swap.


III     Conclusion

We have shown that, under certain assumptions of the swap rate and the future spread of the six-month Libor rate and six-month repo rate, the fair value of the swap spread is given by the expectation of this spread. The spread was given as (5) above.

Market practitioners must set the level of the risk premium factor. Under "normal" market conditions as we move to a market correction or market shock the swap spread will widen. Therefore as the swap rate must be higher when markets are experiencing downside movement, the risk premium will be negative. This gives a higher swap spread at time of market downside correction.

On the basis we have discussed, the swap spread may be used as an alternative benchmark measure used to assess relative value for all debt market instruments. That is, by determining where the market-observed swap spread is relative to that suggested by the fair value swap spread, we can observe the actual over-valuation of benchmark government bonds as captured by the government yield curve.

## References

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