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Learning Curve

Interest Rate Futures Contracts

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The market in short-term interest rate derivatives is a large and liquid one, and the instruments involved are used for a variety of purposes. In this article we review the short-term *interest rate future* contract. Money market derivatives are priced on the basis of the forward rate, and are flexible instruments for hedging against or speculating on forward interest rates. The FRA (see article in "Learning Curve") and exchange-traded interest rate futures contract both date from around the same time, and although initially developed to hedge forward interest rate exposure, they now have a variety of uses. In this article we introduce and analyse the short-term interest rate futures contract.

Forward contracts

A forward contract is an OTC instrument with terms set for delivery of an underlying asset at some point in the future. That is, a forward contract fixes the price and the conditions now for an asset that will be delivered in the future. As each contract is tailor-made to suit user requirements, a forward contract is not as liquid as an exchange-traded futures contract with standardised terms.

The theoretical textbook price of a forward contract is the spot price of the underlying asset plus the funding cost associated with holding the asset until forward expiry date, when the asset is delivered. More formally we may write the price of a forward contract (written on an underlying asset that pays no dividends, such as a zero-coupon bond), as equation (1):

$$P_{fwd}=P_{spot}e^{r} \qquad (1)$$

where

P_{spot}	is the price of the underlying asset of the forward contract;
r	is the continuously compounded risk - free interest rate for a period of maturity n;
n	is the term to maturity of the forward contract in days.

The rule of no-arbitrage pricing states that equation (1) must be true. If $P_{fwd} < P_{und}\, e^{rn}$ then a trader could buy the cheaper instrument, the forward contract, and simultaneously sell the underlying asset. The proceeds from the short sale could be invested at *r* for *n* days; on expiry the short position in the asset is closed out at the forward price *Pfwd* and the trader will have generated a profit of $P_{und}\, e^{rn} - P_{fwd}$. In the opposite scenario, where $P_{fwd} > P_{und}\, e^{rn}$, a trader could put on a long position in the underlying asset, funded at the risk-free interest rate *r* for *n* days, and simultaneously sell the forward contract. On expiry the asset is sold under the terms of the forward contract at the forward price and the proceeds from the sale used to close out the funding initially taken on to buy the asset. Again a profit would be generated, which would be equal to the difference between the two prices.

The relationship described here is used by the market to assume that forward rates implied by the price of short-term interest rate futures contracts are equal to forward rates given by a same-maturity forward contract. Although this assumption holds good for futures contracts with a maturity of up to three or four years, it breaks down for longer-dated futures and forwards.

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Short-term interest rate futures

A *futures* contract is a transaction that fixes the price today for a commodity that will be delivered at some point in the future. Financial futures fix the price for interest rates, bonds, equities and so on, but trade in the same manner as commodity futures. Contracts for futures are standardised and traded on recognised exchanges. In London the main futures exchange is LIFFE, although other futures are also traded on for example, the International Petroleum Exchange and the London Metal Exchange. The money markets trade short-term interest rate futures, which fix the rate of interest on a notional fixed term deposit of money (usually for 90 days or three months) for a specified period in the future. The sum is notional because no actual sum of money is deposited when buying or selling futures; the instrument is off-balance sheet. Buying such a contract is equivalent to making a notional deposit, while selling a contract is equivalent to borrowing a notional sum.
The three-month interest rate future is the most widely used instrument used for hedging interest rate risk.

The LIFFE exchange in London trades short-term interest rate futures for major currencies including sterling, euros, yen and Swiss francs. Table 1 summarises the terms for the short sterling contract as traded on LIFFE.

Table 1: Description of LIFFE short sterling future contract. Source: LIFFE

Name	90-day sterling LIBOR interest rate future
Contract size	£500,000
Delivery months	March, June, September, December
Delivery date	First business day after the last trading day
Last trading day	Third Wednesday of delivery month
Price	100 minus interest rate
Tick size	0.01
Tick value	£12.50
Trading hours	LIFFE CONNECT™ 0805–1800 hours

The original futures contracts related to physical commodities, which is why we speak of *delivery* when referring to the expiry of financial futures contracts. Exchange-traded futures such as those on LIFFE are set to expire every quarter during the year. The short sterling contract is a deposit of cash, so as its price refers to the rate of interest on this deposit, the price of the contract is set as $P = 100 - r$ where P is the price of the contract and r is the rate of interest at the time of expiry implied by the futures contract. This means that if the price of the contract rises, the rate of interest implied goes down, and vice versa. For example the price of the June 1999 short sterling future (written as Jun99 or M99, from the futures identity letters of H, M, U and Z for contracts expiring in March, June, September and December respectively) at the start of trading on 13 March 1999 was 94.880, which implied a three-month LIBOR rate of 5.12% on expiry of the contract in June. If a trader bought 20 contracts at this price and then sold them just before the close of trading that day, when the price had risen to 94.96, an implied rate of 5.04%, she would have made 16 ticks profit or £2000. That is, a 16 tick upward price movement in a long position of 20 contracts is equal to £2000. This is calculated as follows:

$$\text{Profit} = \text{Ticks gained} \times \text{Tick value} \times \text{Number of contracts}$$

$$\text{Loss} = \text{Ticks lost} \times \text{Tick value} \times \text{Number of contracts.}$$

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The tick value for the short sterling contract is straightforward to calculate, since we know that the contract size is £500,000, there is a minimum price movement (tick movement) of 0.005% and the contract has a three-month “maturity”:

$$\text{Tick value} = 0.005\% \times \text{£}500{,}000 \times \frac{3}{12} = \text{£}6.25.$$

The profit made by the trader in our example is logical because if we buy short sterling futures we are depositing (notional) funds; if the price of the futures rises, it means the interest rate has fallen. We profit because we have “deposited” funds at a higher rate beforehand. If we expected sterling interest rates to rise, we would sell short sterling futures, which is equivalent to borrowing funds and locking in the loan rate at a lower level. Note how the concept of buying and selling interest rate futures differs from FRAs: if we buy an FRA we are borrowing notional funds, whereas if we buy a futures contract we are depositing notional funds. If a position in an interest rate futures contract is held to expiry, cash settlement will take place on the delivery day for that contract.

Short-term interest rate contracts in other currencies are similar to the short sterling contract and trade on exchanges such as Eurex in Frankfurt and MATIF in Paris.

Pricing interest rate futures

The price of a three-month interest rate futures contract is the implied interest rate for that currency’s three-month rate at the time of expiry of the contract. Therefore there is always a close relationship and correlation between futures prices, FRA rates (which are derived from futures prices) and cash market rates. On the day of expiry the price of the future will be equal to the LIBOR rate as fixed that day. This is known as the exchange delivery settlement price (EDSP) and is used in the calculation of the delivery amount. During the life of the contract its price will be less closely related to the actual three-month LIBOR rate *today*, but closely related to the *forward rate* for the time of expiry.

Equation (1) was our basic forward rate formula for money market maturity forward rates, which we adapted to use as our FRA price equation. If we incorporate some extra terminology to cover the dealing dates involved it can also be used as our futures price formula. Let us say that:

T_0	is the trade date;
T_M	is the contract expiry date;
T_{CASH}	is the value date for cash market deposits traded on T_0;
T_1	is the value date for cash market deposits traded on T_M;
T_2	is the maturity date for a three-month cash market deposit traded on T_M.

We can then use equation (2) as our futures price formula to obtain *Pfut*, the futures price for a contract up to the expiry date.

$$P_{fut} = 100 - \frac{r_2 n_2 - r_1 n_1}{n_f \left(1 + r_1 \dfrac{n_1}{365}\right)} \qquad (2)$$

where

P_{fut}	is the futures price;
r_1	is the cash market interest rate to T_1;
r_2	is the cash market interest rate to T_2;
n_1	is the number of days from T_{CASH} to T_1;
n_2	is the number of days from T_{CASH} to T_2;
n_f	is the number of days from T_1 to T_2.

The formula uses a 365 day count convention which applies in the sterling money markets; where a market uses a 360-day base this must be used in the equation instead.
In practice the price of a contract at any one time will be close to the theoretical price that would be established by equation (4.6) above. Discrepancies will arise for supply and demand reasons in the market, as well as because LIBOR rates is often quoted only to the nearest sixteenth or 0.0625. The price between FRAs and futures is correlated very closely, in fact banks will often price FRAs using futures, and use futures to hedge their FRA books. When hedging an FRA book with futures, the hedge is quite close to being exact, because the two prices track each other almost tick for tick.[1] However the tick value of a futures contract is fixed, and uses (as we saw above) a 3/12 basis, while FRA settlement values use a 360 or 365-day base. The FRA trader will be aware of this when putting on her hedge.

Forward rates are the market's view on future rates using all information available today. Of course a futures price today is very unlikely to be in line with the actual three-month interest rate that is prevailing at the time of the contract's expiry. This explains why prices for futures and actual cash rates will differ on any particular day. Up until expiry the futures price is the implied forward rate; of course there is always a discrepancy between this forward rate and the cash market rate *today*. The gap between the cash price and the futures price is known as the *basis*. This is defined as:

Basis=Cash price-Futures price.

At any point during the life of a futures contract prior to final settlement – at which point futures and cash rates converge – there is usually a difference between current cash market rates and the rates implied by the futures price. This is the difference we've just explained; in fact the difference between the price implied by the current three-month interbank deposit and the futures price is known as *simple basis*, but it is what most market participants refer to as the basis. Simple basis consists of two separate components, *theoretical basis* and *value basis*. Theoretical basis is the difference between the price implied by the current three-month interbank deposit rate and that implied by the theoretical fair futures price based on cash market forward rates, given by equation (4.6) above. This basis may be either positive or negative depending on the shape of the yield curve.

[1] That is, the basis risk is minimised

The value basis is the difference between the theoretical fair futures price and the actual futures price. It is a measure of how under- or over-valued the futures contract is relative to its fair value. Value basis reflects the fact that a futures contract does not always trade at its mathematically calculated theoretical price, due to the impact of market sentiment and demand and supply. The theoretical and value bases can and do move independently of one another and in response to different influences. Both however converge to zero on the last trading day when final cash settlement of the futures contract is made.

Futures contracts do not in practice provide a precise tool for locking into cash market rates today for a transaction that takes place in the future, although this is what they are in theory designed to do. Futures do allow a bank to lock in a rate for a transaction to take place in the future, and this rate is the *forward rate*. The basis is the difference between today's cash market rate and the forward rate on a particular date in the future. As a futures contract approaches expiry, its price and the rate in the cash market will converge (the process is given the name *convergence*). As we noted earlier this is given by the EDSP and the two prices (rates) will be exactly in line at the exact moment of expiry.

Example 1

The Eurodollar futures contract

The Eurodollar futures contract is traded on the Chicago Mercantile Exchange. The underlying asset is a deposit of US dollars in a bank outside the United States, and the contract is on the rate on dollar 90-day LIBOR. The Eurodollar future is cash settled on the second business day before the third Wednesday of the delivery month (London business day). The final settlement price is used to set the price of the contract, given by:

$$10{,}000\ (100 - 0.25\ r)$$

where r is the quoted Eurodollar rate at the time. This rate is the actual 90-day Eurodollar deposit rate. The longest-dated Eurodollar contract has an expiry date of 10 years. The market assumes that futures prices and forward prices are equal: In practice it also holds for short-dated futures contracts, but is inaccurate for longer-dated futures contracts. Therefore using futures contracts with a maturity greater than five years to calculate zero-coupon rates or implied forward rates will produce errors in results, which need to be taken into account if the derived rates are used to price other instruments such as swaps.

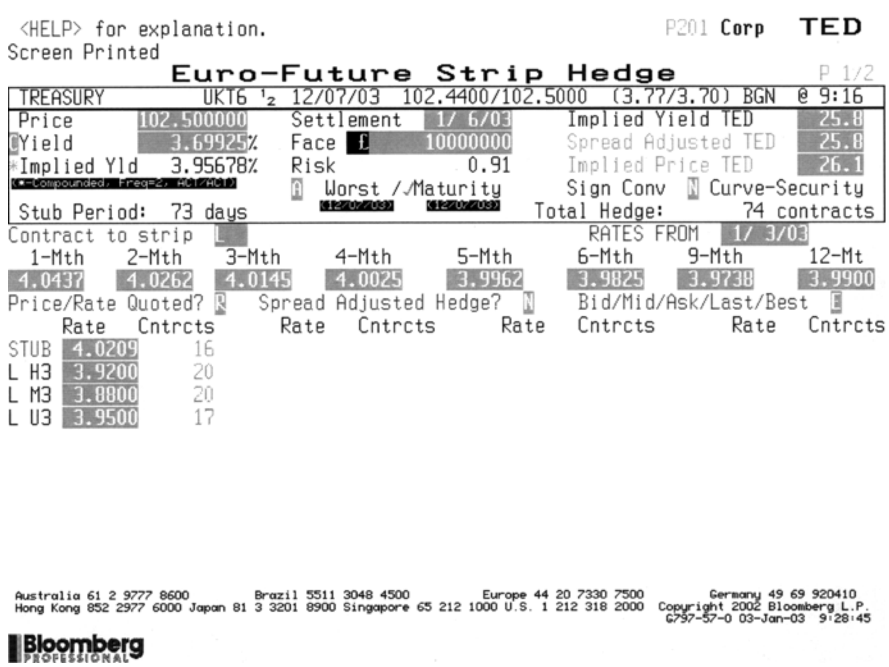
Hedging using interest rate futures

Banks use interest rate futures to hedge interest rate risk exposure in cash and OBS instruments. Bond trading desks also often use futures to hedge positions in bonds of up to two or three years' maturity, as contracts are traded up to three years' maturity. The liquidity of such "far month" contracts is considerably lower than for near month contracts and the "front month" contract (the current contract, for the next maturity month). When hedging a bond with a maturity of say two years, the trader will put on a *strip* of futures contracts that matches as near as possible the expiry date of the bond.

The purpose of a hedge is to protect the value of a current or anticipated cash market or OBS position from adverse changes in interest rates. The hedger will try to offset the effect of the change in interest rate on the value of their cash position with the change in value of their hedging instrument. If the hedge is an exact one the loss on the main position should be compensated by a profit on the hedge position. If the trader is expecting a fall in interest rates and wishes to protect against such a fall they will buy futures, known as a long hedge, and will sell futures (a short hedge) if wishing to protect against a rise in rates.

Bond traders also use three-month interest rate contracts to hedge positions in short-dated bonds; for instance, a market maker running a short-dated bond book would find it more appropriate to hedge his book using short-dated futures rather than the longer-dated bond futures contract. When this happens it is important to accurately calculate the correct number of contracts to use for the hedge. To construct a bond hedge it will be necessary to use a strip of contracts, thus ensuring that the maturity date of the bond is covered by the longest-dated futures contract. The hedge is calculated by finding the sensitivity of each cash flow to changes in each of the relevant forward rates. Each cash flow is considered individually and the hedge values are then aggregated and rounded to the nearest whole number of contracts.

Figure 1 is a reproduction of page TED on a Bloomberg terminal, which calculates the strip hedge for short-dated bonds.[2]. The example shown is for short sterling contract hedge for a position in the UK 6ø% 2003 short-dated gilt, for settlement on 6 January 2003. The screen shows the number of each contract that must be bought (or sold) to hedge the position, which in the example is a holding of £10 million of the bond. The "stub" requirement is met using the near month contract. A total of 73 contracts are required.



[2] This screen was introduced in 1995, the author recalls being in message communication with Bloomberg in Princeton discussing the screen before it went live!

Figure 1: Bloomberg screen TED screen, showing futures strip hedge for position in UK gilt 6.5% 2003, on 3 January 2003 ©Bloomberg LP. Reproduced with permission.

The following examples illustrate hedging with short-term interest rate contracts.

Example 2:Hedging a forward three-month lending requirement

On 1 June a corporate treasurer is expecting a cash inflow of £10 million in three months' time (1 December), which they will then invest for three months. The treasurer expects that interest rates will fall over the next few weeks and wishes to protect themself against such a fall. This can be done using short sterling futures. Market rates on 1 June are as follows:

3- mo LIBOR	6½ %
Sep futures price	93.220

The treasurer buys 20 September short sterling futures at 93.220, this number being exactly equivalent to a sum of £10 million. This allows them to lock in a forward *lending* rate of 6.78%, if we assume there is no bid–offer quote spread.

Expected lending rate = rate implied by futures price
= 100-93.220
= 6.78%

On 1 September market rates are as follows:
3-mo LIBOR 6¼%
Set futures price 93.705
The treasurer unwinds the hedge at this price.
Futures p/l = 97 ticks(93.705 – 93.22) or 0.485%
Effective lending rate = 3-mo LIBOR futures profit
= 6.25% + 0.485%
= 6.735%

The treasurer was quite close to achieving their target lending rate of 6.78% and the hedge has helped to protect against the drop in LIBOR rates from 6ø% to 6÷%, due to the profit from the futures transaction.

In the real world the cash market bid–offer spread will impact the amount of profit/loss from the hedge transaction. Futures generally trade and settle near the offered side of the market rate (LIBOR) whereas lending, certainly by corporates, will be nearer the LIBID rate.

Example 4.5:Hedging a forward six-month borrowing requirement[3]

A treasury dealer has a six-month borrowing requirement for DEM30 million in three months' time, on 16 September. She expects interest rates to rise by at least ø% before that date and would like to lock in a future borrowing rate. The scenario is detailed below.

Date:	16 June
Three-month LIBOR	6.0625%
Six-month LIBOR	6.25
Sep futures contract	93.66
Dec futures contract	93.39

In order to hedge a six-month DEM30 million exposure the dealer needs to use a total of 60 futures contracts, as each has a nominal value of DEM1 million, and corresponds to a three-month notional deposit period. The dealer decides to sell 30 September futures contracts and 30 December futures contracts, which is referred to as a *strip* hedge. The expected forward borrowing rate that can be achieved by this strategy, where the expected borrowing rate is r_f, is calculated as follows:

$$1 + r_f \times \frac{days\,in\,period}{360} = \left(1 + sep\,implied\,rate \times \frac{sep\,days\,period}{360}\right)$$

therefore we have

$$1 + r_f \times \frac{180}{360} = \left(1 + 0.0634 \times \frac{90}{360}\right) \times \left(1 + 0.0661 \times \frac{90}{360}\right) \rightarrow r_f = 6.53\%$$

The rate r_f is sometimes referred to as the "strip rate". The hedge is unwound upon expiry of the September futures contract. Assume the following rates now prevail:

Three-month LIBOR	6.4375%
Six-month LIBOR	6.8125
Sep futures contract	93.56
Dec futures contract	92.93

The futures profit-and-loss is:

September contract:	+10 ticks
December contract:	+46 ticks

This represents a 56 tick or 0.56% profit in three-month interest rate terms, or 0.28% in six-month interest rate terms. The effective borrowing rate is the six-month LIBOR rate minus the futures profit, or:

6.8125% - 0.28% or 6.5325%.

[3] This example predates the introduction of the euro.

In this case the hedge has proved effective because the dealer has realised a borrowing rate of 6.5325%, which is close to the target strip rate of 6.53%. The dealer is still exposed to the basis risk when the December contracts are bought back from the market at the expiry date of the September contract. If for example, the future was bought back at 92.73, the effective borrowing rate would be only 6.4325%, and the dealer would benefit. Of course the other possibility is that the futures contract could be trading 20 ticks more expensive, which would give a borrowing rate of 6.6325%, which is 10 basis points above the target rate. If this happened, the dealer may elect to borrow in the cash market for three months, and maintain the December futures position until the December contract expiry date, and roll over the borrowing at that time. The profit (or loss) on the December futures position will compensate for any change in three-month rates at that time.

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