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Forward Rate Agreements

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In this article we review the *forward rate agreement*. Money market derivatives are priced on the basis of the forward rate, and are flexible instruments for hedging against or speculating on forward interest rates. The FRA and the exchange-traded interest rate future both date from around the same time, and although initially developed to hedge forward interest rate exposure, they now have a variety of uses. In this article the FRA is introduced and analysed, and we review its main uses.

Forward rate agreements

A *forward rate agreement* (FRA) is an OTC derivative instrument that trades as part of the money markets. It is essentially a forward-starting loan, but with no exchange of principal, so that only the difference in interest rates is traded. An FRA is a forward-dated loan, dealt at a fixed rate, but with no exchange of principal – only the interest applicable on the notional amount between the rate dealt and the actual rate prevailing at the time of settlement changes hands. So FRAs are *off-balance sheet* (OBS) instruments. By trading today at an interest rate that is effective at some point in the future, FRAs enable banks and corporates to hedge interest rate exposure. They may also be used to speculate on the level of future interest rates.

Definition

An FRA is an agreement to borrow or lend a *notional* cash sum for a period of time lasting up to twelve months, starting at any point over the next twelve months, at an agreed rate of interest (the FRA rate). The "buyer" of an FRA is borrowing a notional sum of money while the "seller" is lending this cash sum. Note how this differs from all other money market instruments. In the cash market, the party buying a CD or bill, or bidding for stock in the repo market, is the lender of funds. In the FRA market, to "buy" is to "borrow". Of course, we use the term "notional" because with an FRA no borrowing or lending of cash actually takes place, as it is an off-balance sheet product. The notional sum is simply the amount on which interest payment is calculated.

So when an FRA is traded, the buyer is borrowing (and the seller is lending) a specified notional sum at a fixed rate of interest for a specified period, the "loan" to commence at an agreed date in the future. The *buyer* is the notional borrower, and so if there is a rise in interest rates between the date that the FRA is traded and the date that the FRA comes into effect, she will be protected. If there is a fall in interest rates, the buyer must pay the difference between the rate at which the FRA was traded and the actual rate, as a percentage of the notional sum. The buyer may be using the FRA to hedge an actual exposure, that is an actual borrowing of money, or simply speculating on a rise in interest rates. The counterparty to the transaction, the *seller* of the FRA, is the notional lender of funds, and has fixed the rate for lending funds. If there is a fall in interest rates the seller will gain, and if there is a rise in rates the seller will pay. Again, the seller may have an actual loan of cash to hedge or be a speculator.

In FRA trading only the payment that arises as a result of the difference in interest rates changes hands. There is no exchange of cash at the time of the trade. The cash payment that does arise is the difference in interest rates between that at which the FRA was traded and the actual rate prevailing when the FRA matures, as a percentage of the notional amount. FRAs are traded by both banks and corporates and between banks. The FRA market is very liquid in all major currencies and rates are readily quoted on screens by both banks and brokers. Dealing is over the telephone or over a dealing system such as Reuters.

The terminology quoting FRAs refers to the borrowing time period and the time at which the FRA comes into effect (or matures). Hence if a buyer of an FRA wished to hedge against a rise in rates to cover a three-month loan starting in three months' time, she would transact a "three-against-six month" FRA, or more usually a 3 × 6 or 3-v-6 FRA. This is referred to in the market as a "threes-sixes" FRA, and means a three-month loan beginning in three months' time. So a "ones-fours" FRA (1-v-4) is a three-month loan in one month's time, and a "threes-nines" FRA (3-v-9) is six-month money in three months' time.

Note that when one buys an FRA one is "borrowing" funds. This differs from cash products such as CD or repo, as well as interest rate futures, where "buying" is lending funds.

Example 1

A company knows that it will need to borrow £1 million in three months' time for a twelve-month period. It can borrow funds today at LIBOR + 50 basis points. LIBOR rates today are at 5% but the company's treasurer expects rates to go up to about 6% over the next few weeks. So the company will be forced to borrow at higher rates unless some sort of hedge is transacted to protect the borrowing requirement. The treasurer decides to buy a 3-v-15 ("threes-fifteens") FRA to cover the twelve-month period beginning three months from now. A bank quotes 5½% for the FRA which the company buys for a notional £1 million. Three months from now rates have indeed gone up to 6%, so the treasurer must borrow funds at 6½% (the LIBOR rate plus spread), however she will receive a settlement amount which will be the difference between the rate at which the FRA was bought and today's twelve-month LIBOR rate (6%) as a percentage of £1 million, which will compensate for some of the increased borrowing costs.

FRA mechanics

In virtually every market worldwide, FRAs trade under a set of terms and conventions that are identical. The British Bankers' Association (BBA) has compiled standard legal documentation to cover FRA trading. The following standard terms are used in the market.

- **Notional sum**: The amount for which the FRA is traded.
- **Trade date**: The date on which the FRA is dealt.
- **Settlement date**: The date on which the notional loan or deposit of funds becomes effective, that is, is said to begin. This date is used, in conjunction with the notional sum, for calculation purposes only as no actual loan or deposit takes place.
- **Fixing date**: This is the date on which the *reference rate* is determined, that is, the rate to which the FRA dealing rate is compared.
- **Maturity date**: The date on which the notional loan or deposit expires.
- **Contract period**: The time between the settlement date and maturity date.
- **FRA rate**: The interest rate at which the FRA is traded.
- **Reference rate**: This is the rate used as part of the calculation of the settlement amount, usually the LIBOR rate on the fixing date for the contract period in question.
- **Settlement sum**: The amount calculated as the difference between the FRA rate and the reference rate as a percentage of the notional sum, paid by one party to the other on the settlement date.
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These terms are illustrated in Figure 1.

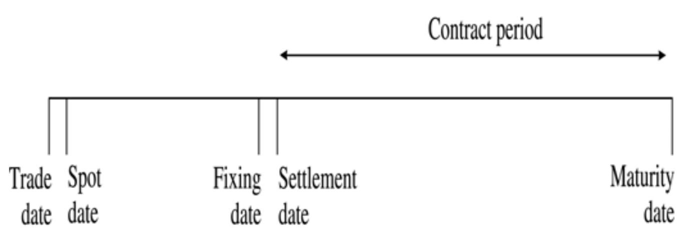

Figure 1: Key dates in an FRA trade

The spot date is usually two business days after the trade date, however it can by agreement be sooner or later than this. The settlement date will be the time period after the spot date referred to by the FRA terms, for example a 1 × 4 FRA will have a settlement date one calendar month after the spot date. The fixing date is usually two business days before the settlement date. The settlement sum is paid on the settlement date, and as it refers to an amount over a period of time that is paid up front, at the start of the contract period, the calculated sum is discounted present value. This is because a normal payment of interest on a loan/deposit is paid at the end of the time period to which it relates; because an FRA makes this payment at the *start* of the relevant period, the settlement amount is a discounted present value sum.

With most FRA trades the reference rate is the LIBOR fixing on the fixing date.

The settlement sum is calculated after the fixing date, for payment on the settlement date. We may illustrate this with a hypothetical example. Consider a case where a corporate has bought £1 million notional of a 1-v-4 FRA, and dealt at 5.75%, and that the market rate is 6.50% on the fixing date. The contract period is 90 days. In the cash market the extra interest charge that the corporate would pay is a simple interest calculation, and is:

$$\frac{6.50-5.75}{100}\times 1{,}000{,}000\times\frac{91}{365}=£1869.$$

This extra interest that the corporate is facing would be payable with the interest payment for the loan, which (as it is a money market loan) is when the loan matures. Under an FRA then, the settlement sum payable should, if it was paid on the same day as the cash market interest charge, be exactly equal to this. This would make it a perfect hedge. As we noted above though, FRA settlement value is paid at the start of the contract period, that is, the beginning of the underlying loan and not the end. Therefore the settlement sum has to be adjusted to account for this, and the amount of the adjustment is the value of the interest that would be earned if the unadjusted cash value was invested for the contract period in the money market. The settlement value is given by equation (1)

$$Settlement=\frac{\left(r_{ref}-r_{FRA}\right)\times M\times\frac{n}{B}}{1+\left(r_{ref}\times\frac{n}{B}\right)} \tag{1}$$

where

- r_{ref} is the reference interest fixing rate;
- r_{FRA} is the FRA rate or contract rate;
- M is the notional value;
- n is the number of days in the contract period;
- B is the day-count base (360 or 365).

Equation (1) simply calculates the extra interest payable in the cash market, resulting from the difference between the two interest rates, and then discounts the amount because it is payable at the start of the period and not, as would happen in the cash market, at the end of the period.

In our hypothetical illustration, as the fixing rate is higher than the dealt rate, the corporate buyer of the FRA receives the settlement sum from the seller. This then compensates the corporate for the higher borrowing costs that he would have to pay in the cash market. If the fixing rate had been lower than 5.75%, the buyer would pay the difference to the seller, because the cash market rates will mean that he is subject to a lower interest rate in the cash market. What the FRA has done is hedge the interest rate, so that whatever happens in the market, it will pay 5.75% on its borrowing.

A market maker in FRAs is trading short-term interest rates. The settlement sum is the value of the FRA. The concept is exactly as with trading short-term interest rate futures; a trader who buys an FRA is running a long position, so that if on the fixing date $r_{ref} > r_{FRA}$, the settlement sum is positive and the trader realises a profit. What has happened is that the trader, by buying the FRA, "borrowed" money at an interest rate, which subsequently rose. This is a gain, exactly like a *short* position in an interest rate future, where if the price goes down (that is, interest rates go up), the trader realises a gain. Equally a "short" position in an FRA, put on by selling an FRA, realises a gain if on the fixing date $r_{ref} < r_{FRA}$.

FRA pricing

As their name implies, FRAs are forward rate instruments and are priced using forward rate principles. Consider an investor who has two alternatives, either a six-month investment at 5% or a one-year investment at 6%. If the investor wishes to invest for six months and then roll over the investment for a further six months, what rate is required for the rollover period such that the final return equals the 6% available from the one-year investment? If we view an FRA rate as the breakeven forward rate between the two periods, we simply solve for this forward rate and that is our approximate FRA rate. This rate is sometimes referred to as the interest rate "gap" in the money markets (not to be confused with an interbank desk's *gap risk*, the interest rate exposure arising from the net maturity position of its assets and liabilities).

We can use the standard forward-rate breakeven formula to solve for the required FRA rate; we established this relationship earlier when discussing the calculation of forward rates that are arbitrage-free. The relationship given in equation (2) connects simple (bullet) interest rates for periods of time up to one year, where no compounding of interest is required. As FRAs are money market instruments we are not required to calculate rates for periods in excess of one year,[1] where compounding would need to be built into the equation. This is given by equation (2)

$$(1 + r_2 t_2) = (1 + r_1 t_1)(1 + r_f t_f) \tag{2}$$

where

- r_2 is the cash market interest rate for the long period;
- r_1 is the cash market interest rate for the short period;
- r_f is the forward rate for the gap period;
- t_2 is the time period from today to the end of the long period;
- t_1 is the time period from today to the end of the short period;
- t_f is the forward gap time period, or the contract period for the FRA.

This is illustrated diagrammatically in Figure 2.

[1] Although it is of course possible to trade FRAs with contract periods greater than one year, for which a different pricing formula must be used.

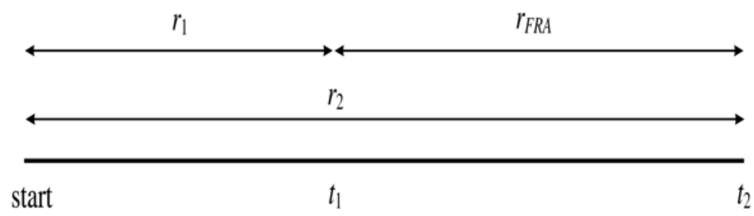

Figure 2: Rates used in FRA pricing

The time period t_1 is the time from the dealing date to the FRA settlement date, while $t2$ is the time from the dealing date to the FRA maturity date. The time period for the FRA (contract period) is t_2 minus t_1. We can replace the symbol t for time period with n for the actual number of days in the time periods themselves. If we do this and then rearrange the equation to solve for r_{FRA} the FRA rate, we obtain (3):

$$r_{FRA} = \frac{r_2 n_2 - r_1 n_1}{n_{fra}\left(1 + r_1 \frac{n_1}{365}\right)} \tag{3}$$

where

n_1	is the number of days from the dealing date or spot date to the settlement date;
n_2	is the number of days from the dealing date or spot date to the maturity date;
r_1	is the spot rate to the settlement date;
r_2	is the spot rate from the spot date in the FRA contract period;
n_{fra}	is the number of days in the maturity date;
r_{FRA}	is the FRA rate.

If the formula is applied to (say) the US money markets, the 365 in the equation is replaced by 360, the day count base for that market.

In practice FRAs are priced off the exchange-traded short-term interest rate future for that currency, so that sterling FRAs are priced off LIFFE short sterling futures. Traders normally use a spreadsheet pricing model that has futures prices directly fed into it. FRA positions are also usually hedged with other FRAs or short-term interest rate futures.

Example 2:Hedging an FRA position

An FRA market maker sells a EUR 100 million 3-v-6 FRA, that is, an agreement to make a notional deposit (without exchange of principal) for three months in three months' time, at a rate of 7.52%. He is exposed to the risk that interest rates will have risen by the FRA settlement date in three months' time.

Date	14 December
3-v-6 FRA rate	7.52%
March futures price	92.50%
Current spot rate	6.85%

Action

The dealer first needs to calculate a precise hedge ratio. This is a three-stage process:

1. Calculate the nominal value of a basis point move in LIBOR on the FRA settlement payment;

$$BPV = FRA_{nom} \times 0.01\% \times \frac{n}{360}.$$

Therefore: €100,000,000 $\times$ 0.01% $\times$ 90/360= €2500.

2. Find the present value of 1. By discounting it back to the transaction date using the FRA and spot rates;

Present value of a basis point move=

$$\frac{\text{Nominal value of basis point}}{\left(1+\text{spot rate}\times\frac{\text{Days in hedge period}}{360}\right)\times\left(1+\text{FRA rate}\times\frac{\text{Days in hedge period}}{360}\right)}.$$

Therefore:

$$\frac{€2500}{\left(1+6.85\%\times\frac{90}{360}\right)\times\left(1+7.52\%\times\frac{90}{360}\right)}$$

3. Determine the correct hedge ratio by dividing 2 by the futures tick value.

$$\text{Hedge ratio} = \frac{2412}{25} = 96.48.$$

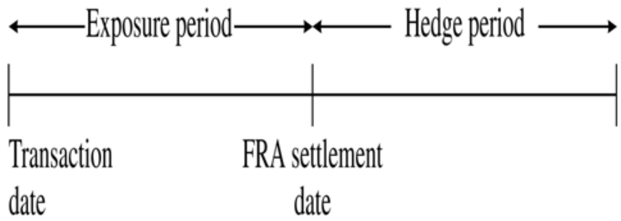

Figure3: FRA dates

The appropriate number of contracts for the hedge of a EUR 100,000,000 3-v-6 FRA would therefore be 96 or 97, as the fraction is under one-half, 96 is correct. To hedge the risk of an increase in interest rates, the trader sells 96 ECU three months' futures contracts at 92.50. Any increase in rates during the hedge period should be offset by a gain realised on the futures contracts through daily variation margin receipts.

Outcome

Date	15 March
Three month LIBOR	7.625%
March EDSP	92.38

The hedge is lifted upon expiry of the March futures contracts. Three-month LIBOR on the FRA settlement date has risen to 7.625% so the trader incurs a loss of EUR 25,759 on his FRA position (i.e., EUR 26,250 discounted back over the three month FRA period at current LIBOR rate), calculated as follows:

$$\frac{(\text{LIBOR-FRA rate})\times(\text{days in FRA period}/360)\times\text{Contract Nominal Amount}}{1+\text{LIBOR rate}\times(\text{days in FRA period}/360)}$$

Therefore:

$$\frac{26,250^{*}}{1+7.625\%\times\frac{90}{360}}=\text{EUR } 25,759$$

*i.e., $0.105\%\times\frac{90}{360}\times\text{Eur}\,100,000,000$

Futures P/L: 12 ticks (92.50-92.38)$\times$ €25$\times$96 contracts = EUR 28,800.

The EUR 25,759 loss on the FRA position is more than offset by the EUR 28,800 profit on the futures position when the hedge is lifted. If the dealer has sold 100 contracts his futures profit would have been EUR 30,000, and, accordingly, a less accurate hedge. The excess profit in the hedge position can mostly be attributed to the arbitrage profit realised by the market maker (i.e., the market maker has sold the FRA for 7.52% and in effect bought it back in the futures market by selling futures at 92.50 or 7.50% for a 2 tick profit.)