Learning Curve

Using Bond Futures Contracts for Trading and Hedging

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A widely used risk management instrument in the debt capital markets is the government bond futures contract. This is an exchange-traded standardised contract that fixes the price today at which a specified quantity and quality of a bond will be delivered at a date during the expiry month of the futures contract. Unlike short-term interest rate futures, which only require cash settlement, bond futures require the actual physical delivery of a bond when they are settled.

In this article we review bond futures contracts and their use for trading and hedging purposes.

Introduction

A *futures contract* is an agreement between two counterparties that fixes the terms of an exchange that will take place between them at some future date. They are standardised agreements as opposed to OTC ones, when traded on an exchange, so they are also referred to as *exchange traded futures*. In the UK financial futures are traded on LIFFE, the London International Financial Futures Exchange which opened in 1982. LIFFE is the biggest financial futures exchange in Europe in terms of volume of contracts traded. There are four classes of contract traded on LIFFE: short-term interest rate contracts, long-term interest rate contracts (bond futures), currency contracts and stock index contracts.

We discussed short-term interest rate futures contracts, which generally trade as part of the money markets, in the June 2004 Learning Curve . Here we will look at bond futures contracts, which are an important part of the bond markets; they are used for hedging and speculative purposes. Most futures contracts on exchanges around the world trade at three-month maturity intervals, with maturity dates fixed at March, June, September and December each year. This includes the contracts traded on LIFFE. Therefore at pre-set times during the year a contract for each of these months will *expire*, and a final *settlement* price is determined for it. The further out one goes the less liquid the trading is in that contract. It is normal to see liquid trading only in the *front month* contract (the current contract, so that if we are trading in April 2002 the front month is the June 2002 future), and possibly one or two of the next contracts, for most bond futures contracts. The liquidity of contracts diminishes the further one trades out in the maturity range.

When a party establishes a position in a futures contract, it can either run this position to maturity or close out the position between trade date and maturity. If a position is closed out the party will have either a profit or loss to book. If a position is held until maturity, the party who is long futures will take delivery of the underlying asset (bond) at the settlement price; the party who is short futures will deliver the underlying asset. This is referred to as *physical settlement* or sometimes, confusingly, as *cash settlement*.

There is no counterparty risk associated with trading exchange-traded futures, because of the role of the *clearing house*, such as the London Clearing House. This is the body through which contracts are settled. A clearing house acts as the buyer to all contracts sold on the exchange, and the seller to all contracts that are bought. So in the London market the LCH acts as the counterparty to all transactions, so that settlement is effectively guaranteed. The LCH requires all exchange participants to deposit *margin* with it, a cash sum that is the cost of conducting business (plus brokers' commissions). The size of the margin depends on the size of a party's net *open* position in contracts (an open position is a position in a contract that is held overnight and not closed out). There are two types of margin, *maintenance margin* and *variation margin*. Maintenance margin is the minimum level required to be held at the clearing house; the level is set by the exchange. Variation margin is the additional amount that must be deposited to cover any trading losses and as the size of the net open positions increases. Note that this is not like margin in say, a repo transaction. Margin in repo is a safeguard against a drop in value of collateral that has been supplied against a loan of cash. The margin deposited at a futures exchange clearing house acts essentially as "good faith" funds, required to provide comfort to the exchange that the futures trader is able to satisfy the obligations of the futures contract.

Contract specifications

We have noted that futures contracts traded on an exchange are standardised. This means that each contract represents exactly the same commodity, and it cannot be tailored to meet individual customer requirements. In this section we describe two very liquid and commonly traded contracts, starting with the US T-bond contract traded on the Chicago Board of Trade (CBOT). The details of this contract are given in Table 1.

Unit of trading	US Treasury bond with notional value of $100,000 and a coupon of 8%
Deliverable grades	US T-bonds with a minimum maturity of 15 years from first day of delivery month
Delivery months	March, June, September, December
Delivery date	Any business day during the delivery month
Last trading day	12:00 noon, seventh business day before last business day of delivery month
Quotation	Per cent of par expressed as points and thirty-seconds of a point, e.g., 108 − 16 is 108 16/32 or 108.50
Minimum price movement	1/32
Tick value	$31.25
Trading hours	07:20–14:00 (trading pit) 17:20–20:05 22:30–06:00 hours (screen trading)

Table 1: CBOT US T-bond futures contract. Source: CBOT

The terms of this contract relate to a US Treasury bond with a minimum maturity of 15 years and a *notional* coupon of 8%. We introduced the concept of the notional bond in the chapter on repo markets. A futures contract specifies a notional coupon to prevent delivery and liquidity problems that would arise if there was shortage of bonds with exactly the coupon required, or if one market participant purchased a large proportion of all the bonds in issue with the required coupon. For exchange-traded futures, a short future can deliver any bond that fits the maturity criteria specified in the contract terms. Of course a long future would like to deliver a high-coupon bond with significant accrued interest, while the short future would want to deliver a low-coupon bond with low interest accrued. In fact this issue does not arise because of the way the *invoice amount* (the amount paid by the long future to purchase the bond) is calculated. The invoice amount on the expiry date is given as equation (1):

$$Inv_{amt} = P_{fut} \times CF + AI \qquad (1)$$

where

Inv_{amt} is the invoice amount;
P_{fut} is the price of the futures contract;
CF is the conversion factor;
AI is the bond accrued interest.

Any bond that meets the maturity specifications of the futures contract is said to be in the *delivery basket*, the group of bonds that are eligible to be delivered into the futures contract. Every bond in the delivery basket will have its own *conversion factor*, which is used to equalise coupon and accrued interest differences of all the delivery bonds. The exchange will announce the conversion factor for each bond before trading in a contract begins; the conversion factor for a bond will change over time, but remains fixed for one individual contract. That is, if a bond has a conversion factor of 1.091252, this will remain fixed for the life of the contract. If a contract specifies a bond with a notional coupon of 7%, like the long gilt future on LIFFE, then the conversion factor will be less than 1.0 for bonds with a coupon lower than 7% and higher than 1.0 for bonds with a coupon higher than 7%. A formal definition of conversion factor is given below.

Conversion factor

The conversion factor (or price factor) gives the price of an individual cash bond such that its yield to maturity on the delivery day of the futures contract is equal to the notional coupon of the contract. The product of the conversion factor and the futures price is the forward price available in the futures market for that cash bond (plus the cost of funding, referred to as the gross basis).

Although conversion factors equalise the yield on bonds, bonds in the delivery basket will trade at different yields, and for this reason they are not “equal” at the time of delivery. Certain bonds will be cheaper than others, and one bond will be the *cheapest-to-deliver* bond. The cheapest-to-deliver bond is the one that gives the greatest return from a strategy of buying a bond and simultaneously selling the futures contract, and then closing out positions on the expiry of the contract. This so-called *cash-and-carry trading* is actively pursued by proprietary trading desks in banks. If a contract is purchased and then held to maturity the buyer will receive, via the exchange’s clearing house the cheapest-to-deliver gilt. Traders sometimes try to exploit arbitrage price differentials between the future and the cheapest-to-deliver gilt, known as *basis trading*.

We summarise the contract specification of the long gilt futures contract traded on LIFFE in Table 2. There is also a medium gilt contract on LIFFE, which was introduced in 1998 (having been discontinued in the early 1990s). This trades a notional five-year gilt, with eligible gilts being those of four to seven years’ maturity.

Unit of trading	UK gilt bond having a face value of £100,000, a notional coupon of 7% and a notional maturity of 10 years (changed from contract value of £50,000 from the September 1998 contract)
Deliverable grades	UK gilts with a maturity ranging from 8¾ to 13 years from the first day of the delivery month (changed from 10–15 years from the December 1998 contract)
Delivery months	March, June, September, December
Delivery date	Any business day during the delivery month
Last trading day	11:00 hours, two business days before last business day of delivery month
Quotation	Per cent of par expressed as points and hundredths of a point, for example 114.56 (changed from ticks and 1/32nds of a point, as in 117−17 meaning 114 17/32 or 114.53125, from the June 1998 contract)
Minimum price movement	0.01 of one point (one tick)
Tick value	£10
Trading hours	08:00–18:00 hours All trading conducted electronically on LIFFE CONNECT™ platform

Table 2: LIFFE long gilt future contract specification. Source: LIFFE

Futures pricing

Theoretical principle

Although it may not appear so on first trading, floor trading on a futures exchange is probably the closest one gets to an example of the economist's perfect and efficient market. The immediacy and liquidity of the market will ensure that at virtually all times the price of any futures contract reflects fair value. In essence because a futures contract represents an underlying asset, albeit a synthetic one, its price cannot differ from the actual cash market price of the asset itself. This is because the market sets futures prices such that they are arbitrage-free. We can illustrate this with a hypothetical example.

Let us say that the benchmark 10-year bond, with a coupon of 8% is trading at par. This bond is the underlying asset represented by the long bond futures contract; the front month contract expires in precisely three months. If we also say that the three-month LIBOR rate (the repo rate) is 6%, what is fair value for the front month futures contract?

For the purpose of illustration let us start by assuming the futures price to be 105. We could carry out the following arbitrage-type trade:

- buy the bond for £100;
- simultaneously sell the future at £105;
- borrow £100 for three months at the repo rate of 6%.

As this is a leveraged trade we have borrowed the funds with which to buy the bond, and the loan is fixed at three months because we will hold the position to the futures contract expiry, which is in exactly three months' time. At expiry, as we are short futures we will deliver the underlying bond to the futures clearing house and close out the loan. This strategy will result in cash flows for us as shown below.

Futures settlement cash flows		
Price received for bond	=	105.00
Bond accrued	=	2.00 (8% coupon for three months)
Total proceeds	=	107.00
Loan cash flows		
Repayment of principal	=	100.00
Loan interest	=	1.500 (6% repo rate for three months)
Total outlay	=	101.50

The trade has resulted in a profit of £5.50, and this profit is guaranteed as we have traded the two positions simultaneously and held them both to maturity. We are not affected by subsequent market movements. The trade is an example of a pure arbitrage, which is risk-free. There is no cash outflow at the start of the trade because we borrowed the funds used to buy the bond. In essence we have locked in the forward price of the bond by trading the future today, so that the final settlement price of the futures contract is irrelevant. If the situation described above were to occur in practice it would be very short-lived, precisely because arbitrageurs would buy the bond and sell the future to make this profit. This activity would force changes in the prices of both bond and future until the profit opportunity was removed.

So in our illustration the price of the future was too high (and possibly the price of the bond was too low as well) and not reflecting fair value because the price of the synthetic asset was out of line with the cash asset.

What if the price of the future was too low? Let us imagine that the futures contract is trading at 95.00. We could then carry out the following trade:

- sell the bond at £100;
- simultaneously buy the future for £95;
- lend the proceeds of the short sale (£100) for three months at 6%.

This trade has the same procedure as the first one with no initial cash outflow, except that we have to cover the short position in the repo market, through which we invest the sale proceeds at the repo rate of 6%. After three months we are delivered a bond as part of the futures settlement, and this is used to close out our short position. How has our strategy performed?

Futures settlement cash flows
Clean price of bond=95.00
Bond accrued = 2.00
Total cash outflow = 97.00
Loan cash flows
Principal on loan maturity = 100.00
Interest from loan = 1.500
Total cash inflow = 101.500

The profit of £4.50 is again a risk-free arbitrage profit. Of course our hypothetical world has ignored considerations such as bid–offer spreads for the bond, future and repo rates, which would apply in the real world and impact on any trading strategy. Yet again however the futures price is out of line with the cash market and has provided opportunity for arbitrage profit.

Given the terms and conditions that apply in our example, there is one price for the futures contract at which no arbitrage profit opportunity is available. If we set the future price at 99.5, we would see that both trading strategies, buying the bond and selling the future or selling the bond and buying the future, yield a net cash flow of zero. There is no profit to be made from either strategy. So at 99.5 the futures price is in line with the cash market, and it will only move as the cash market price moves; any other price will result in an arbitrage profit opportunity.

Arbitrage-free futures pricing

The previous section demonstrated how we can arrive at the fair value for a bond futures contract provided we have certain market information. The market mechanism and continuous trading will ensure that the fair price *is* achieved, as arbitrage profit opportunities are eliminated. We can determine the bond future's price given:

- the coupon of the underlying bond, and its price in the cash market;
- the interest rate for borrowing or lending funds, from the trade date to the maturity date of the futures contract. This is known as the *repo* rate.

For the purpose of deriving this pricing model we can ignore bid–offer spreads and borrowing and lending spreads. We set the following:

r is the repo rate;
rc is the bond's running yield;
P_{bond} is the price of the cash bond;
P_{fut} is the price of the futures contract;
t is the time to the expiry of the futures contract.

We can substitute these symbols into the cash flow profile for our earlier trade strategy, that of buying the bond and selling the future. This gives us:

Futures settlement cash flows
Clean price for bond $= P_{fut}$
Bond accrued $= rc \times t \times P_{bond}$
Total proceeds $= P_{fut} + (rc \times t \times P_{bond})$

Loan cash flows
Repayment of loan principal $=$ P_{bond}
Loan interest $=$ $r \times t \times P_{bond}$
Total outlay $=$ $P_{bond} + (r \times t \times P_{bond})$

The profit from the trade would be the difference between the proceeds and outlay, which we can set as follows:

$$\text{Profit} = P_{fut} + rc \times t \times P_{bond} - P_{bond} + r \times t \times P_{bond}. \qquad (2)$$

We have seen how the futures price is at fair value when there is no profit to be gained from carrying out this trade, so if we set profit at zero, we obtain the following:

$$0 = P_{fut} + rc \times t \times P_{bond} - P_{bond} + r \times t \times P_{bond}$$

Solving this expression for the futures price P_{fut} gives us.

$$P_{fut} = P_{bond} + P_{bond}\, t(r - rc).$$

Rearranging this we get:

$$P_{fut} = P_{bond}\,(1+\ t(\,r - rc)). \qquad (3)$$

If we repeat the procedure for the other strategy, that of selling the bond and simultaneously buying the future, and set the profit to zero, we will obtain the same equation for the futures price as given in equation (3).

It is the level of the repo rate in the market, compared to the running yield on the underlying bond, that sets the price for the futures contract. From the examples used at the start of this section we can see that it is the cost of funding compared to the repo rate that determines if the trade strategy results in a profit. The equation ($r - rc$) from (3) is the net financing cost in the arbitrage trade, and is known as the *cost of carry*. If the running yield on the bond is higher than the funding cost (the repo rate) this is positive funding or *positive carry*. Negative funding (*negative carry*) is when the repo rate is higher than the running yield. The level of (r - rc) will determine whether the futures price is trading above the cash market price or below it. If we have positive carry (when rc > r) then the futures price will trade below the cash market price, known as trading at a *discount*. Where r > rc and we have negative carry then the futures price will be at a premium over the cash market price. If the net funding cost was zero, such that we had neither positive nor negative carry, then the futures price would be equal to the underlying bond price.

The cost of carry related to a bond futures contract is a function of the yield curve. In a positive yield curve environment the three-month repo rate is likely to be lower than the running yield on a bond so that the cost of carry is likely to be positive. As there is generally only a liquid market in long bond futures out to contracts that mature up to one year from the trade date, with a positive yield curve it would be unusual to have a short-term repo rate higher than the running yield on the long bond. So in such an environment we would have the future trading at a discount to the underlying cash bond. If there is a negative sloping yield curve the futures price will trade at a premium to the cash price. It is in circumstances of changes in the shape of the yield curve that opportunities for relative value and arbitrage trading arise, especially as the bond that is cheapest-to-deliver for the futures contract may change with large changes in the curve.

A trading strategy that involved simultaneous and opposite positions in the cheapest-to-deliver bond (CTD) and the futures contract is known as *cash-and-carry* trading or *basis trading*. However, by the law of no-arbitrage pricing, the payoff from such a trading strategy should be zero. If we set the profit from such a trading strategy as zero, we can obtain a pricing formula for the fair value of a futures contract, which summarises the discussion above, and states that the fair value futures price is a function of the cost of carry on the underlying bond. This is given as equation (4):

$$P_{fut} = \frac{(P_{bond} + AI_0) \times (1 + rt) - \sum_{i=1}^{N} C_i (1 + rt_{i,del}) - AI_{del}}{CF} \qquad (4)$$

where

AI_0 is the accrued interest on the underlying bond today;
AI_{del} is the accrued interest on the underlying bond on the expiry or delivery date (assuming the bond is delivered on the final day, which will be the case if the running yield on the bond is above the money market rate);
C_i is the *i*th coupon;
N is the number of coupons paid from today to the expiry or delivery date;
r is the repo rate;
t is the time period (in years) over which the trade takes place;
CF is the bond conversion factor;
$t_{i,del}$ is the period from receipt of the *i*th coupon to delivery.

Hedging using bond futures

Introduction

Bond futures are used for a variety of purposes. Much of one day's trading in futures will be speculative, that is, a punt on the direction of the market. Another main use of futures is to hedge bond positions. In theory if hedging a cash bond position with a bond futures contract, if cash and futures prices move together then any loss from one position will be offset by a gain from the other. When prices move exactly in lock-step with each other, the hedge is considered perfect. In practice the price of even the cheapest-to-deliver bond (which one can view as being the bond being traded – implicitly – when one is trading the bond future) and the bond future will not move exactly in line with each other over a period of time. The difference between the cash price and the futures price is called the *basis*. The risk that the basis will change in an unpredictable way is known as *basis risk*.

Futures are a liquid and straightforward way of hedging a bond position. By hedging a bond position the trader or fund manager is hoping to balance the loss on the cash position by the profit gained from the hedge. However the hedge will not be exact for all bonds except the cheapest-to-deliver (CTD) bond, which we can assume is the futures contract underlying bond. The basis risk in a hedge position arises because the bond being hedged is not identical to the CTD bond. The basic principle is that if the trader is long (or net long, where the desk is running long and short positions in different bonds) in the cash market, an equivalent number of futures contracts will be sold to set up the hedge. If the cash position is short the trader will buy futures. The hedging requirement can arise for different reasons. A market maker will wish to hedge positions arising out of client business, when they are unsure when the resulting bond positions will be unwound. A fund manager may, for example, know that they need to realise a cash sum at a specific time in the future to meet fund liabilities, and sell bonds at that time. The market maker will want to hedge against a drop in value of positions during the time the bonds are held. The fund manager will want to hedge against a rise in interest rates between now and the bond sale date, to protect the value of the portfolio.

When putting on the hedge position the key is to trade the correct number of futures contracts. This is determined by using the *hedge ratio* of the bond and the future, which is a function of the volatilities of the two instruments. The amount of contracts to trade is calculated using the hedge ratio, which is given by:

$$Hedge\ ratio = \frac{Volatility\ of\ bond\ to\ be\ hedged}{Volatility\ of\ hedging\ instrument}.$$

Therefore one needs to use the volatility values of each instrument. We can see from the calculation that if the bond is more volatile than the hedging instrument, then a greater amount of the hedging instrument will be required. Let us now look in greater detail at the hedge ratio.

There are different methods available to calculate hedge ratios. The most common ones are the conversion factor method, which can be used for deliverable bonds (also known as the *price factor* method) and the modified duration method (also known as the basis point value method).

Where a hedge is put on against a bond that is in the futures delivery basket it is common for the conversion factor to be used to calculate the hedge ratio. A conversion factor hedge ratio is more useful as it is transparent and remains constant, irrespective of any changes in the price of the cash bond or the futures contract. The number of futures contracts required to hedge a deliverable bond using the conversion factor hedge ratio is determined using equation (5):

$$Number\ of\ contracts = \frac{M_{bond} \times CF}{M_{fut}} \tag{5}$$

where M is the nominal value of the bond or futures contract.

The conversion factor method may only be used for bonds in the delivery basket. It is important to ensure that this method is only used for one bond. It is an erroneous procedure to use the ratio of conversion factors of two different bonds when calculating a hedge ratio. This will be considered again later.

Unlike the conversion factor method, the modified duration hedge ratio may be used for all bonds, both deliverable and non-deliverable. In calculating this hedge ratio the modified duration is multiplied by the dirty price of the cash bond to obtain the *basis point value* (BPV). The BPV represents the actual impact of a change in the yield on the price of a specific bond. The BPV allows the trader to calculate the hedge ratio to reflect the different price sensitivity of the chosen bond (compared to the CTD bond) to interest rate movements. The hedge ratio calculated using BPVs must be constantly updated, because it will change if the price of the bond and/or the futures contract changes. This may necessitate periodic adjustments to the number of lots used in the hedge.

The number of futures contracts required to hedge a bond using the BPV method is calculated using the following:

$$\text{Number of contracts} = \frac{M_{bond}}{M_{fut}} \times \frac{BPV_{bond}}{BPV_{fut}} \tag{6}$$

where the BPV of a futures contract is defined with respect to the BPV of its CTD bond, as given by equation (7):

$$BPV_{fut} = \frac{BPV_{CTDbond}}{CF_{CTDbond}} \tag{7}$$

The simplest hedge procedure to undertake is one for a position consisting of only one bond, the cheapest-to-deliver bond. The relationship between the futures price and the price of the CTD given by equation (4) indicates that the price of the future will move for moves in the price of the CTD bond, so therefore we may set:

$$\Delta P_{fut} \simeq \frac{\Delta P_{Bond}}{CF} \tag{8}$$

where *CF* is the CTD conversion factor.

The price of the futures contract, over time, does not move tick-for-tick with the CTD bond (although it may on an intra-day basis) but rather by the amount of the change divided by the conversion factor. It is apparent therefore that to hedge a position in the CTD bond we must hold the number of futures contracts equivalent to the value of bonds held multiplied by the conversion factor. Obviously if a conversion factor is less than one, the number of futures contracts will be less than the equivalent nominal value of the cash position; the opposite is true for bonds that have a conversion factor greater than one. However the hedge is not as simple as dividing the nominal value of the bond position by the nominal value represented by one futures contract.

To measure the effectiveness of the hedge position, it is necessary to compare the performance of the futures position with that of the cash bond position, and to see how much the hedge instrument mirrored the performance of the cash instrument. A simple calculation is made to measure the effectiveness of the hedge, given by equation (9), which is the percentage value of the hedge effectiveness:

$$\text{Hedege effectiveness} = -\left(\frac{Fut\ p/l}{Bond\ p/l}\right) \times 100. \tag{9}$$

Hedging a bond portfolio

The principles established above may be applied when hedging a portfolio containing a number of bonds. It is more realistic to consider a portfolio holding not just bonds that are outside the delivery basket, but are also not government bonds. In this case we need to calculate the number of futures contracts to put on as a hedge based on the volatility of each bond in the portfolio compared to the volatility of the CTD bond. Note that in practice, there is usually more than one futures contract that may be used as the hedge instrument. For example in the sterling market it would be more sensible to use LIFFE's medium gilt contract, whose underlying bond has a notional maturity of four to seven years, if hedging a portfolio of short- to medium-dated bonds. However for the purposes of illustration we will assume that only one contract, the long bond, is available.

To calculate the number of futures contracts required to hold as a hedge against any specific bond, we use equation (10).

$$Hedge = \frac{M_{bond}}{M_{fut}} \times Vol_{bond/CTD} \times Vol_{CTD/fut} \tag{10}$$

where

M is the nominal value of the bond or future;

$Vol_{bond/CTD}$ is the relative volatility of the bond being hedged compared to that of the CTD bond;

$Vol_{CTD/fut}$ is the relative volatility of the CTD bond compared to that of the future.

It is not necessarily straightforward to determine the relative volatility of a bond vis-à-vis the CTD bond. If the bond being hedged is a government bond, we can calculate the relative volatility using the two bonds' modified duration. This is because the yields of both may be safely assumed to be strongly positively correlated. If however the bond being hedged is a corporate bond and/or non-vanilla bond, we must obtain the relative volatility using regression analysis, as the yields between the two bonds may not be strongly positively correlated. This is apparent when one remembers that the yield spread of corporate bonds over government bonds is not constant, and will fluctuate with changes in government bond yields. To use regression analysis to determine relative volatilities, historical price data on the bond is required; the daily price moves in the target bond and the CTD bond are then analysed to assess the slope of the regression line. In this section we will restrict the discussion to a portfolio of government bonds.

If we are hedging a portfolio of government bonds we can use equation (11) to determine relative volatility values, based on the modified duration of each of the bonds in the portfolio.

$$\text{Vol}_{\text{bond/CTD}} = \frac{\triangle P_{bond}}{\triangle P_{CTD}} \approx \frac{MD_{bond} \times P_{bond}}{MD_{CTD} \times P_{CTD}} \tag{11}$$

where *MD* is the modified duration of the bond being hedged or the CTD bond, as appropriate.[1]

Once we have calculated the relative volatility of the bond being hedged, equation (12) (obtained from (8) and (11)) tells us that the relative volatility of the CTD bond to that of the futures contract is approximately the same as its conversion factor. We are then in a position to calculate the futures hedge for each bond in a portfolio.

$$\text{Vol}_{\text{CTD/fut}} = \frac{\triangle P_{CTD}}{\triangle P_{fut}} \approx CF_{CTD} \tag{12}$$

Table3 shows a portfolio of five UK gilts on 20 October 1999. The nominal value of the bonds in the portfolio is £200 million, and the bonds have a market value excluding accrued interest of £206.84 million. Only one of the bonds is a deliverable bond, the 5 ¾% 2009 gilt which is in fact the CTD bond. For the Dec ’99 futures contract the bond had a conversion factor of 0.9124950. The fact that this bond is the CTD explains why it has a relative volatility of 1. We calculate the number of futures contracts required to hedge each position, using the equations listed above. For example, the hedge requirement for the position in the 7% 2002 gilt was calculated as follows:

$$\frac{5,000,000}{100,000} \times \frac{2.245 \times 101.50}{7.235 \times 99.84} \times 0.9124950 = 14.39.$$

The volatility of all the bonds is calculated relative to the CTD bond, and the number of futures contracts determined using the conversion factor for the CTD bond. The bond with the highest volatility is not surprisingly the 6% 2028, which has the longest maturity of all the bonds and hence the highest modified duration. We note from Table 5.3 that the portfolio requires a hedge position of 2091 futures contracts. This illustrates how a “rough-and-ready” estimate of the hedging requirement, based on nominal values, would be insufficient as that would suggest a hedge position of only 2000 contracts.

[1] In certain textbooks and practitioner research documents, it is suggested that the ratio of the conversion factors of the bond being hedged (if it is in the delivery basket) and the CTD bond can be used to determine the relative volatility of the target bond. This is a specious argument. The conversion factor of a deliverable bond is the price factor that will set the yield of the bond equal to the notional coupon of the futures contract on the delivery date, and it is a function mainly of the coupon of the deliverable bond. The price volatility of a bond on the other hand, is a measure of its modified duration, which is a function of the bond’s duration (that is, the weighted average term to maturity). Therefore using conversion factors to measure volatility levels will produce erroneous results. It is important not to misuse conversion factors when arranging hedge ratios.

CTD		5.75% 2009	Modified duration			7.234565567	
Conversion factor		0.9124950	Price			99.84	
Bond	Nominal amount (£m)	Price	Yield %	Duration	Modified duration	Relative volatility	Number of contr-acts
UKT 8% 2000	12	102.17	5.972	1.072	1.011587967	0.143090242	15.67
UKT 7% 2002	5	101.50	6.367	2.388	2.245057208	0.315483336	14.39
UKT 5% 2004	38	94.74	6.327	4.104	3.859791022	0.50626761	175.55
UKT 5.75% 2009	100	99.84	5.770	7.652	7.234565567	1.00	912.50
UKT 6% 2028	45	119.25	4.770	15.031	14.34666412	2.368603078	972.60
Total	200						2090.71

Table 3: Bond futures hedge for hypothetical gilt portfolio, 20 October 1999

The effectiveness of the hedge must be monitored over time. No hedge will be completely perfect however, and the calculation illustrated above, as it uses modified duration value, does not take into account the convexity effect of the bonds. The reason why a futures hedge will not be perfect is because in practice, the price of the futures contract will not move tick-for-tick with the CTD bond, at least not over a period of time. This is the basis risk that is inherent in hedging cash bonds with futures. In addition, the calculation of the hedge is only completely accurate for a parallel shift in yields, as it is based on modified duration, so as the yield curve changes around pivots, the hedge will move out of line. Finally, the long gilt future is not the appropriate contract to use to hedge three of the bonds in the portfolio, or over 25% of the portfolio by nominal value. This is because these bonds are short- or medium-dated, and so their price movements will not track the futures price as closely as longer-dated bonds. In this case, the more appropriate futures contract to use would have been the medium gilt contract, or (for the first bond, the 8% 2000) a strip of short sterling contracts. Using shorter-dated instruments would reduce some of the basis risk contained in the portfolio hedge.

The margin process

Institutions buying and selling futures on an exchange deal with only one counterparty at all times, the exchange clearing house. The clearing house is responsible for the settlement of all contracts, including managing the delivery process. A central clearing mechanism eliminates counterparty risk for anyone dealing on the exchange, because the clearing house guarantees the settlement of all transactions. The clearing house may be owned by the exchange itself, such as the one associated with the Chicago Mercantile Exchange (the CME Clearinghouse) or it may be a separate entity, such as the London Clearing House, which settles transactions on LIFFE. The LCH is also involved in running clearing systems for swaps and repo products in certain currencies.

One of the key benefits to the market of the clearing house mechanism is that counterparty risk, as it is transferred to the clearing house, is virtually eliminated. The mechanism that enables the clearing house to accept the counterparty risk is the *margining* process that is employed at all futures exchanges. A bank or local trader must deposit margin before commencing dealing on the exchange; each day a further amount must be deposited or will be returned, depending on the results of the day's trading activity.

The exchange will specify the level of margin that must be deposited for each type of futures contract that a bank wishes to deal in. The *initial margin* will be a fixed sum per lot, so for example if the margin was £1000 per lot an opening position of 100 lots would require margin of £100,000. Once initial margin has been deposited, there is a mark-to-market of all positions at the close of business; exchange-traded instruments are the most transparent products in the market, and the closing price is not only known to everyone, it is also indisputable. The closing price is also known as the *settlement price*. Any losses suffered by a trading counterparty, whether closed out or run overnight, are entered as a debit on the party's account and must be paid the next day. Trading profits are credited and may be withdrawn from the margin account the next day. This daily process is known as *variation margining*. Thus the margin account is updated on a daily basis and the maximum loss that must be made up on any morning is the maximum price movement that occurred the previous day. It is a serious issue if a trading party is unable to meet a margin call. In such a case, the exchange will order it to cease trading, and will also liquidate all its open positions; any losses will be met out of the firm's margin account. If the level of funds in the margin account is insufficient, the losses will be made good from funds paid out of a general fund run by the clearing house, which is maintained by all members of the exchange.

Payment of margin is made by electronic funds transfer between the trading party's bank account and the clearing house. Initial margin is usually paid in cash, although clearing houses will also accept high-quality securities such as T-bills or certain government bonds, to the value of the margin required. Variation margin is always cash. The advantage of depositing securities rather than cash is that the depositing firm earns interest on its margin. This is not available on a cash margin, and the interest foregone on a cash margin is effectively the cost of trading futures on the exchange. However if securities are used, there is effectively no cost associated with trading on the exchange (we ignore of course infrastructure costs and staff salaries).

The daily settlement of exchange-traded futures contracts, as opposed to when the contract expires or the position is closed out, is the main reason why futures prices are not equal to forward prices for long-dated instruments.

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